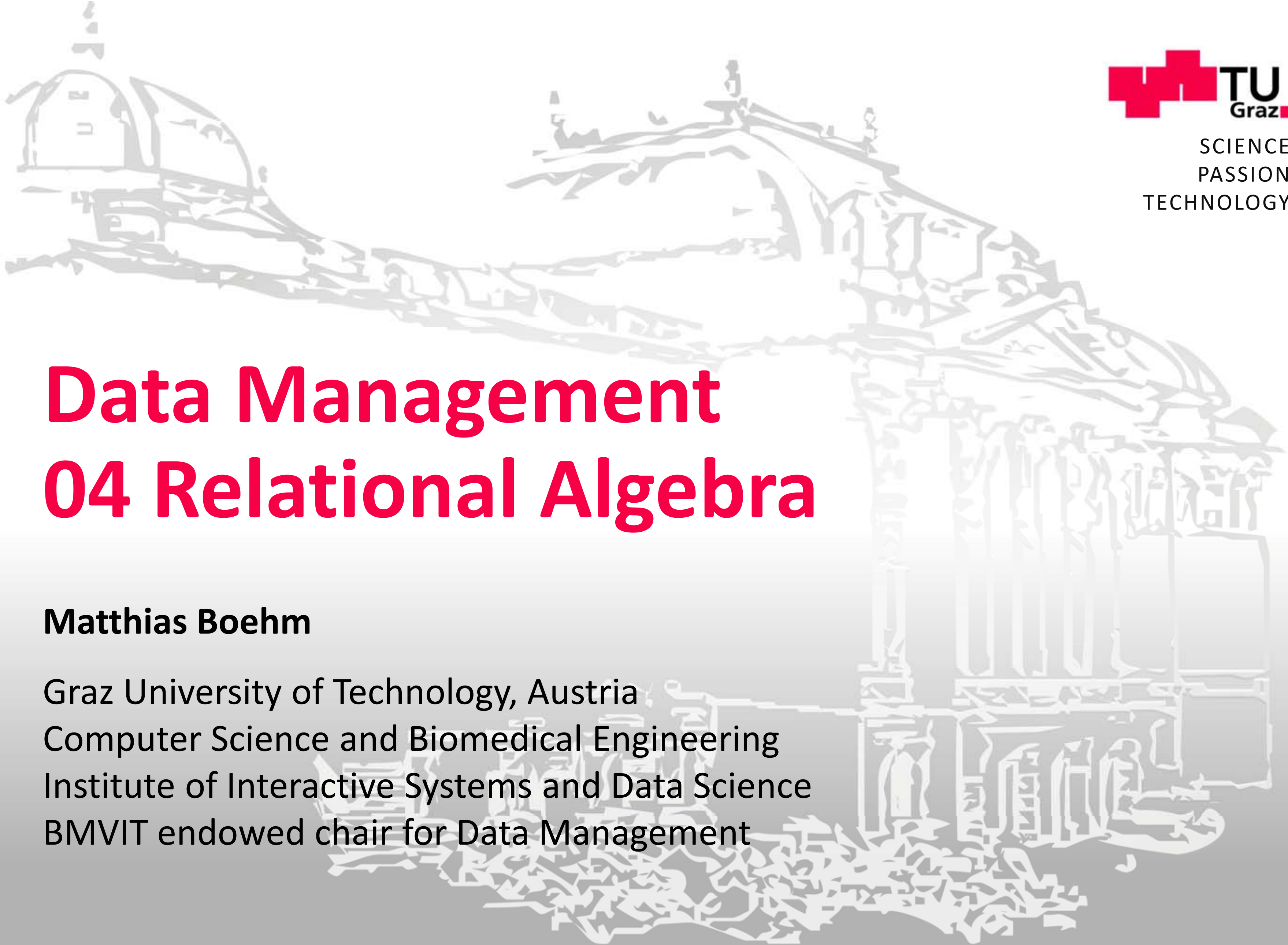

SCIENCE
PASSION
TECHNOLOGY

Data Management
04 Relational Algebra

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BMVIT endowed chair for Data Management

Last update: Oct 28, 2019

Announcements/Org

- **#1 Video Recording**
  - Link in TeachCenter & TUbe (lectures will be public)
- **#2 Info Study Abroad**
  - 5-10min in lecture **today** (start 6.10pm)
- **#3 Exercise Submission**
  - Submission through TeachCenter (max 5MB, draft possible)
  - Open since Oct 15 (deadline **Nov 05, 11.59pm**)

➔ **Nobody left behind** (if problems, contact us via newsgroup or email)

POP-UP INFORMATION BOOTH - OCTOBER 2019

Mobility Programmes for students

- exchange programmes
- summer programmes
- internships
- research

we care about international education

Recap: Relations and Terminology

- **Domain D (value domain): e.g., Set S, INT, Char[20]**
- **Relation R**
  - **Relation schema** RS:
    Set of k attributes $\{A_1,\ldots,A_k\}$
  - **Attribute** A_j: value domain $D_j = \mathrm{dom}(A_j)$
  - **Relation:** subset of the Cartesian product over all value domains D_j
    $\mathbf{R \subseteq D_1 \times D_2 \times \ldots \times D_k,\ k \geq 1}$

Attribute

A1 INT	A2 INT	A3 BOOL
3	7	T
1	2	T
3	4	F
1	7	T

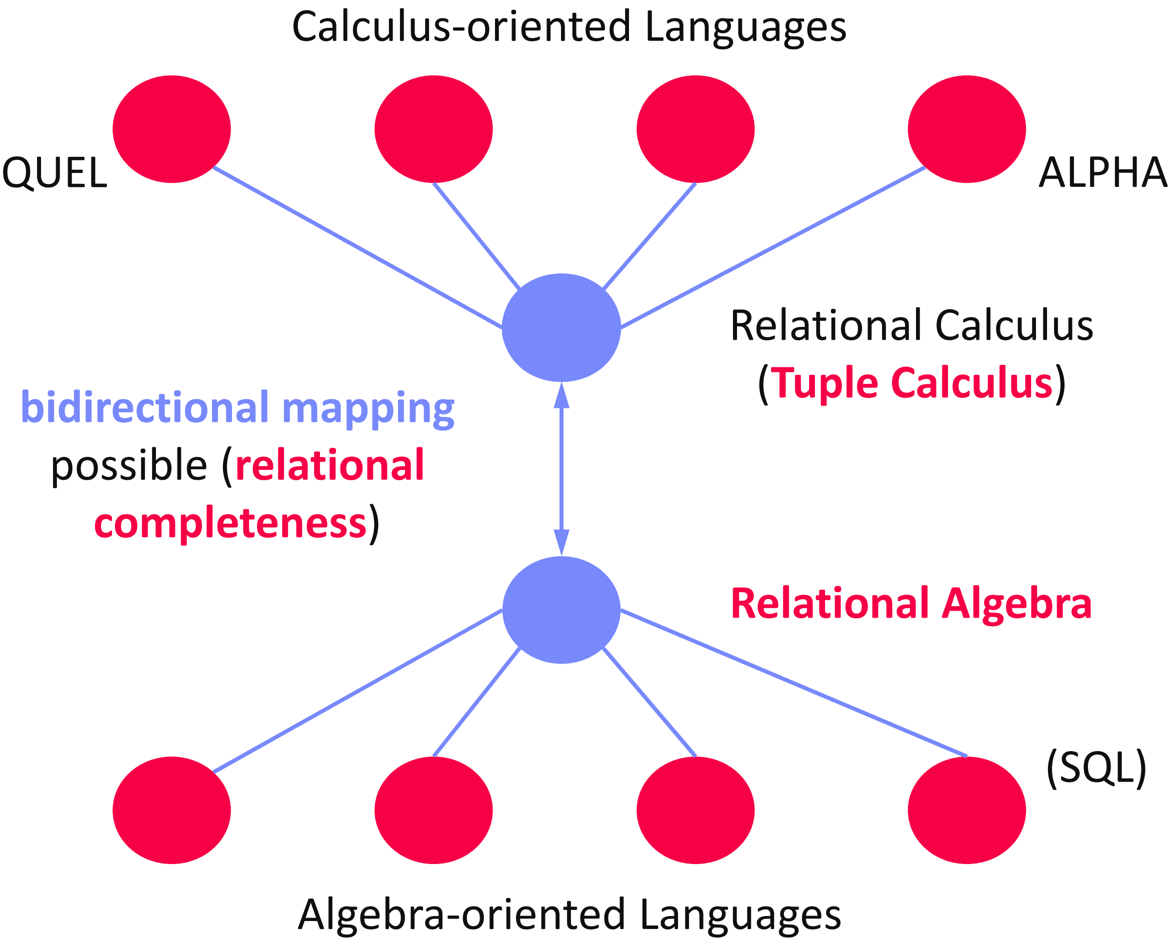
Tuple

cardinality: 4
rank: 3

- **Additional Terminology**
  - **Tuple**: row of k elements of a relation
  - **Cardinality** of a relation: number of tuples in the relation
  - **Rank** of a relation: number of attributes
  - **Semantics:** **Set** := no duplicate tuples (in practice: **Bag** := duplicates allowed)
  - **Order of tuples and attributes is irrelevant**

Relational Algebra vs Tuple Calculus

- **Comparison Scheme for Data Sub Languages**



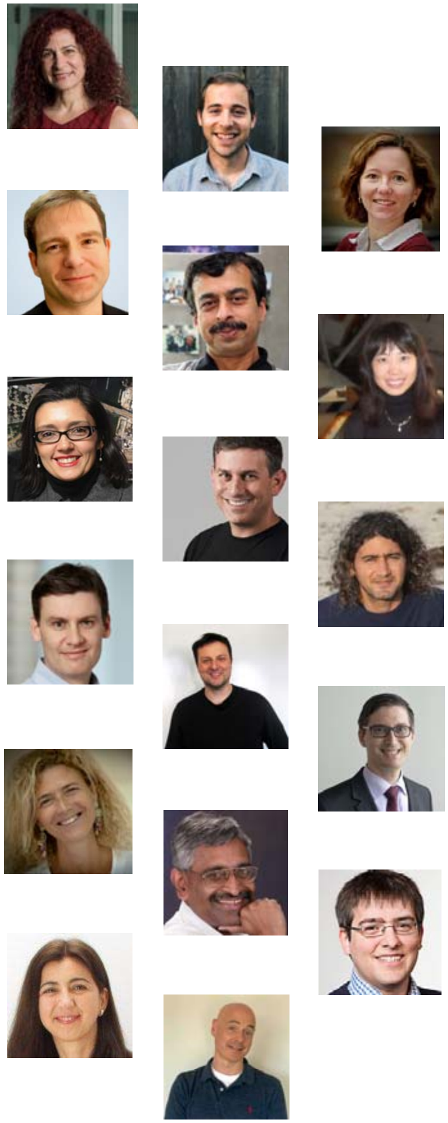
[E. F. Codd: Relational Completeness of Data Base Sublanguages. IBM Research Report RJ987, **1972**]

Agenda

- **Relational Algebra**
- **Tuple Calculus**
- **Physical Operators** (Preview Lecture 08)

Database Research Self-Assessment 2018

PID	Firstname	Lastname	Affiliation	LID
102	Anastasia	Ailamaki	EPFL	1
104	Peter	Bailis	Stanford	
105	Magdalena	Balazinska	U Washington	3
107	Peter	Boncz	CWI	2
108	Surajit	Chaudhuri	MS Research	3
111	Luna	Dong	Amazon	3
113	Juliana	Freire	NYU	5
115	Joe	Hellerstein	UC Berkley	6
116	Stratos	Idreos	Harvard	7
117	Donald	Kossman	MS Research	
118	Tim	Kraska	MIT	7
120	Volker	Markl	TU Berlin	8
122	Tova	Milo	Tel Aviv University	9
123	C.	Mohan	IBM Research	10
124	Thomas	Neumann	TU Munich	11
126	Fatma	Ozcan	IBM Research	10
130	Christopher	Re	Stanford	4



Database Research Self-Assessment 2018, cont.

PID	…	Affiliation	LID
102		EPFL	1
104		Stanford	
105		U Washington	3
107		CWI	2
108		MS Research	3
111		Amazon	3
113		NYU	5
115		UC Berkley	6
116		Harvard	7
117		MS Research	
118		MIT	7
120		TU Berlin	8
122		Tel Aviv University	9
123		IBM Research	10
124		TU Munich	11
126		IBM Research	10
130		Stanford	4

LID	Location
1	Lausanne, SUI
2	Amsterdam, NLD
3	Seattle, USA
4	Stanford, USA
5	New York, USA
6	Berkley, USA
7	Cambridge, USA
8	Berlin, GER
9	Tel Aviv, ISR
10	San Jose, USA
11	Munich, GER

Relational Algebra

Core Relational Algebra

- **Relational Algebra**
  - **Operands: relations** (normalized, variables for computing new values)
  - **Operators**: traditional **set operations** and specific **relational operations**
- **Basic Operations** (minimal)
  - Cartesian product $R \times S$
  - Union $R \cup S$
  - Difference $R – S$
  - Projection $\pi_{i_1, \dots, i_m}(R)$
  - Selection $\sigma_F(R)$
- **Derived Operations**
  - Intersection $R \cap S$
  - Join $R \bowtie S$
  - Division $R \div S$
- **Rename $\rho_S(R)$**

	Traditional **Set Operations**	Specific **Relational Operations**
Basic Operations	$R \times S$ $R \cup S$ $R – S$	$\pi_{i_1, \dots, i_m}(R)$ $\sigma_F(R)$
Derived Operations	$R \cap S$	$R \bowtie S$ $R \div S$

Extended Relational Algebra

- **Extended Relational Algebra**
  - Relational algebra introduced with **set semantics** (no duplicate tuples)
  - SQL with **bag semantics** (more flexibility and performance)
  - **Codd'72:** *In a practical environment it would need to be augmented by a counting and summing capability, together with […] library functions […].*
- **Additional Operations (Ext)**
  - Duplicate elimination **$\delta(R)$**
  - Grouping **$\gamma_{A,f(B)}R$**
  - Sorting **$\tau_A(R)$**

- **Bag (aka Multiset) Terminology**
  - Multiplicity: # occurrences of an instance
  - Cardinality: # tuples (i.e., # instances weighted by multiplicity)

A	B
a	b
b	c
a	b

Cartesian Product

Basic
Derived
Ext

- **Definition: R×S := {(r,s) | r ∈ R, s ∈ S}**
  - Set of all pairs of inputs (equivalent in set/bag)

- **Example**

PID	Firstname	Lastname	Affiliation	LID
104	Peter	Bailis	Stanford	
130	Christopher	Re	Stanford	4

×

SF Bay Area

LID	Location
4	Stanford, USA
6	Berkley, USA
10	San Jose, USA

PID	Firstname	Lastname	Affiliation	LID	LID	Location
104	Peter	Bailis	Stanford		4	Stanford, USA
130	Christopher	Re	Stanford	4	4	Stanford, USA
104	Peter	Bailis	Stanford		6	Berkley, USA
130	Christopher	Re	Stanford	4	6	Berkley, USA
104	Peter	Bailis	Stanford		10	San Jose, USA
130	Christopher	Re	Stanford	4	10	San Jose, USA

Union

Basic
Derived
Ext

- **Definition:** $R \cup S := \{x \mid x \in R \lor x \in S\}$
  - **Set:** set union with duplicate elimination (idempotent: $S \cup S = S$)
  - **Bag:** bag union (commutative but not idempotent)

- **Example w/ set semantics**

Firstname	Lastname	Affiliation
Anastasia	Ailamaki	EPFL
Peter	Boncz	CWI
Volker	Markl	TU Berlin
Thomas	Neumann	TU Munich

Firstname	Lastname	Affiliation
Anastasia	Ailamaki	EPFL
Magdalena	Balazinska	U Washington
Juliana	Freire	NYU
Tova	Milo	Tel Aviv University

Firstname	Lastname	Affiliation
Anastasia	Ailamaki	EPFL
Peter	Boncz	CWI
Volker	Markl	TU Berlin
Thomas	Neumann	TU Munich
Magdalena	Balazinska	U Washington
Juliana	Freire	NYU
Tova	Milo	Tel Aviv University

Union requires
compatible schema

Difference

Basic | Ext
Derived

- **Definition: R–S := {x | x ∈ R ∧ x ∉ S}** (sometimes \)
  - **Set:** set difference
  - **Bag:** element multiplicity of R minus multiplicity min(R,S)

- **Example w/ bag semantics**

Affiliation
Stanford
U Washington
MS Research
Amazon
NYU
UC Berkley
Harvard
MIT
IBM Research

–

Affiliation
Stanford
U Washington
NYU
UC Berkley
Harvard
MIT
TU Berlin
Stanford

➡

Affiliation
MS Research
Amazon
IBM Research

Difference requires **compatible schema**

Projection and Selection

Basic
Derived
Ext

- **Projection $\pi_{i_1, \ldots, i_m}(R)$**
  - **Set:** selection of attributes with duplicate elimination
  - **Bag:** selection of attributes
- **Extended Projection**
  - Arithmetic expressions: $\pi_{A,A*B}(R)$
  - Duplicate occurrences: $\pi_{A,A,B}(R)$
- **Selection (restriction) $\sigma_F(R)$**
  - Selection of tuples satisfying the predicate F (equivalent in **set**/**bag**)
  - Example: $\sigma_{Affiliation=\text{‘IBM Research‘}}(R)$

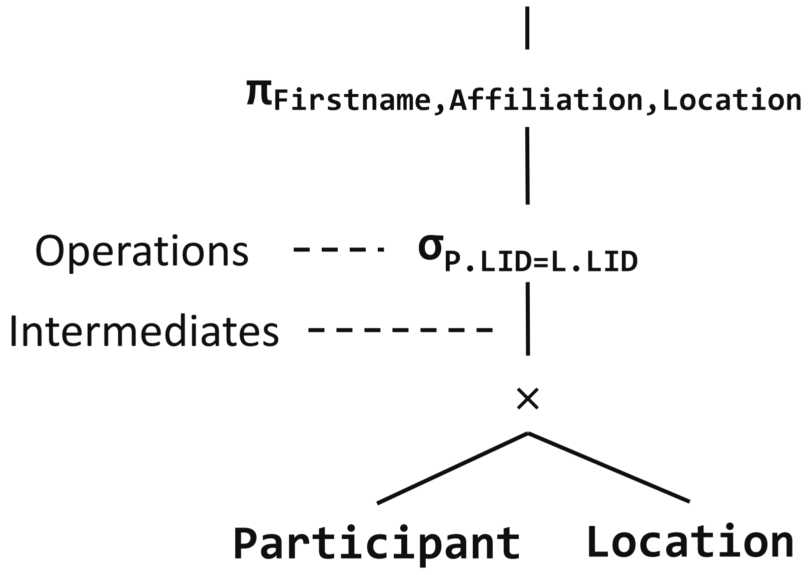
PID	Firstname	Lastname	Affiliation	LID
123	C.	Mohan	IBM Research	10
126	Fatma	Ozcan	IBM Research	10

Example:
$\pi_{Affiliation}(R)$ w/
set semantics

Affiliation
EPFL
Stanford
U Washington
CWI
MS Research
Amazon
NYU
UC Berkley
Harvard
MIT
TU Berlin
Tel Aviv University
IBM Research
TU Munich

Query Trees

- **Composition of Complex Queries**
  - Relational algebra expressions → **data flow graph (tree)**
  - Leaf nodes represent base relations; root node represent result
- **Example** $\pi_{\texttt{Firstname,Affiliation,Location}}(\sigma_{\texttt{P.LID=L.LID}}(\texttt{Participant} \times \texttt{Location}))$



Firstname	Affiliation	Location
Anastasia	EPFL	Lausanne, SUI
Magdalena	U Washington	Seattle, USA
Peter	CWI	Amsterdam, NLD
Surajit	MS Research	Seattle, USA
…	…	…

Intersection

Basic | Derived | Ext

- **Definition R∩S :=** $\boxed{R - (R - S)}$ (derived from basic operations)
  - **Set:** set intersection derived from difference
  - **Bag:** bag intersection, with element multiplicity min(R,S)
- **Example**

Firstname	Lastname	Affiliation
Anastasia	Ailamaki	EPFL
Peter	Boncz	CWI
Volker	Markl	TU Berlin
Thomas	Neumann	TU Munich

Firstname	Lastname	Affiliation
Anastasia	Ailamaki	EPFL
Magdalena	Balazinska	U Washington
Juliana	Freire	NYU
Tova	Milo	Tel Aviv University

➡

Firstname	Lastname	Affiliation
Anastasia	Ailamaki	EPFL

Intersection requires **compatible schema**

Division

Basic
Derived
Ext

- **Definition** $R \div S := \pi_{R-S}(R) - \pi_{R-S}((\pi_{R-S}(R) \times S) - R)$
  - Find instances in R that satisfy S (e.g., **which students took ALL DB course**)
  - $R \div S := \{(a_1, \ldots, a_{r-s}) \mid \forall (b_1, \ldots, b_s) \in S : (a_1, \ldots, a_{r-s}, b_1, \ldots, b_s) \in R\}$

- **Example**

R

A	B	C	D
a	b	c	d
a	b	e	f
b	c	e	f
e	d	e	f
e	d	c	d
a	b	d	e

S

C	D
c	d
e	f

(many-to-one
set containment test)

R÷S

A	B
a	b
e	d

- **Example Derivation**

$\pi_{R-S}(R)$

A	B
a	b
b	c
e	d

$\pi_{R-S}(R) \times S$

A	B	C	D
a	b	c	d
a	b	e	f
b	c	c	d
b	c	e	f
e	d	c	d
e	d	e	f

$(\pi_{R-S}(R) \times S) - R$

A	B	C	D
b	c	c	d

$\pi_{R-S}((\pi_{R-S}(R) \times S) - R)$

A	B
b	c

$\pi_{R-S}(R) - \pi_{R-S}((\pi_{R-S}(R) \times S) - R)$

A	B
a	b
e	d

Join

Basic | Ext
Derived

- **Definition $R \bowtie S := \pi ... (\sigma_F (R \times S))$**
  - Selection of tuples (and attributes) from the catesian product R×S (equivalent in set/bag); **beware of NULLs:** do never match
  - **Theta Join:** $R \bowtie_\Theta S := \sigma_\Theta (R \times S)$; arbitrary condition e.g., $\Theta \in \{=, \neq, <, \leq, >, \geq\}$
  - **Natural Join:** $R \bowtie S$; **equi join** (Θ is =) w/ shared attributes appearing once

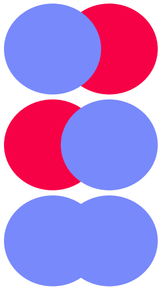
- **Example Natural Join Participant ⋈ Location**

<u>PID</u>	Firstname	Lastname	Affiliation	LID
102	Anastasia	Ailamaki	EPFL	1
104	Peter	Bailis	Stanford	
105	Magdalena	Balazinska	U Washington	3

➡

PID	Firstname	Lastname	Affiliation	LID	Location
102	Anastasia	Ailamaki	EPFL	1	Lausanne, SUI
105	Magdalena	Balazinska	U Washington	3	Seattle, USA

Types of Joins



- **Outer Joins**
  - Left outer join ⟕ (tuples of lhs, NULLs for non-existing rhs)
  - Right outer join ⟖ (tuples of rhs, NULLs for non-existing lhs)
  - Full outer join ⟗ (tuples of lhs/rhs, NULLs for non-existing lhs/rhs)
  - Example

Participant ⟕ Location

PID	Firstname	Lastname	Affiliation	LID	LID	Location
102	Anastasia	Ailamaki	EPFL	1	1	Lausanne, SUI
104	Peter	Bailis	Stanford	NULL	NULL	NULL
105	Magdalena	Balazinska	UW	3	3	Seattle, USA

- **Semi Join**
  - Left semi join $\ltimes := \pi_R(R \bowtie S)$ (filter lhs)
  - Right semi join ⋊ (filter rhs)
  - Example Participant $\ltimes\ \sigma_{GER}$ (Location)

PID	Firstname	Lastname	Affiliation	LID
120	Volker	Markl	TU Berlin	8
124	Thomas	Neumann	TU Munich	11

- **Anti Join**
  - Left anti join $R \rhd S := R - R \ltimes S$ (**complement of left semi join**)
  - Right anti join (complement of right semi join)

Deduplication, Sorting, and Renaming

Basic
Derived
Ext

- **Duplication Elimination $\delta(R)$**
  - Convert a bag into a **set** by removing all duplicate instances
  - SQL: use **ALL** or **DISTINCT** to indicate w/ or w/o duplicate elimination

- **Sorting $\tau_A(R)$**
  - Convert a bag into a **sorted list** of tuples; order lost if used in other ops
  - SQL: sequence of attributes with ASC (ascending) or DESC (descending) order
  - Example: $\tau_{\text{Firstname ASC, Lastname ASC}}(\text{Participant})$

- **Rename $\rho_S(R)$**
  - Define new schema (attribute names), but keep tuples unchanged
  - **Example:** $\rho_{\text{ID, Given Name, Family Name, Affiliation, LID}}(\text{Participant})$

Grouping and Aggregation

Basic Derived Ext

- **Definition $\gamma_{A,f(B)}R$**
  - Grouping: group input tuples R according to unique values in A
  - Aggregation: compute aggregate f(B) per group of tuples (aggregation w/o grouping possible)
- **Example $\gamma_{Affiliation,COUNT(*)}Participant$**

Affiliation	COUNT
EPFL	1
Stanford	2
U Washington	1
CWI	1
MS Research	2
Amazon	1
…	…
IBM Research	2
TU Munich	1

- **Classification of Aggregates f(B)**
  - Additive aggregation functions (SUM, COUNT)
  - Semi-additive aggregation functions (MIN, MAX)
  - Additively computable aggregation functions (AVG, STDDEV, VAR)
  - Aggregation functions (MEDIAN, QUANTILES)

BREAK (and Test Yourself)

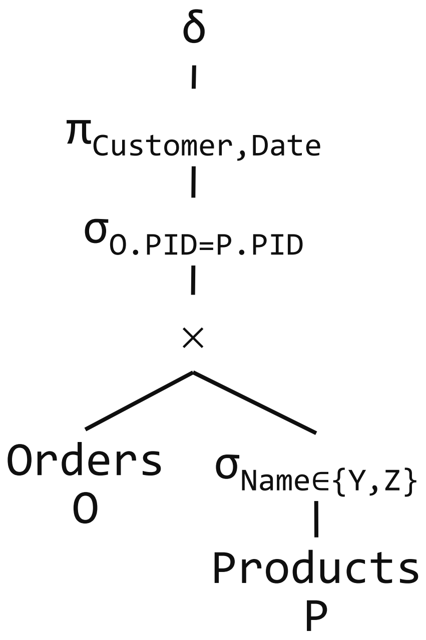
- **Task: Compute the results for the following queries.**

Orders

OID	Customer	Date	Quantity	PID
1	A	‘2019-06-22’	3	2
2	B	‘2019-06-22’	1	3
3	A	‘2019-06-22’	1	4
4	C	‘2019-06-23’	2	2
5	D	‘2019-06-23’	1	4
6	C	‘2019-06-23’	1	1

Products

PID	Name	Price
1	X	100
2	Y	15
4	Z	75
3	W	120



δ

$\pi_{\text{Customer,Date}}$

$\sigma_{\text{O.PID=P.PID}}$

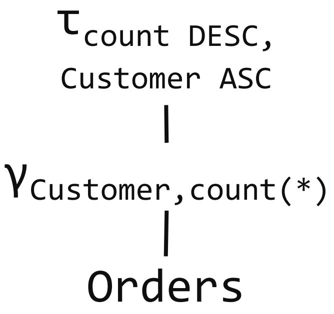
$\times$

Orders O

$\sigma_{\text{Name}\in\{\text{Y,Z}\}}$

Products P

Customer	Date
A	‘2019-06-22’
C	‘2019-06-23’
D	‘2019-06-23’

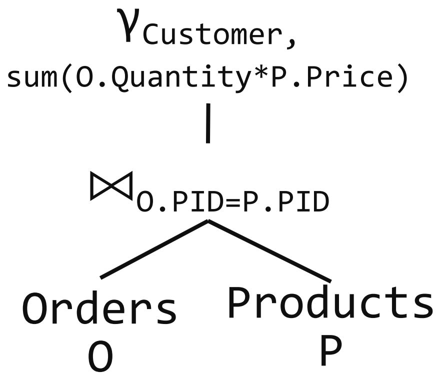


$\tau_{\text{count DESC, Customer ASC}}$

$\gamma_{\text{Customer,count(*)}}$

Orders

Customer	Count
A	2
C	2
B	1
D	1



$\gamma_{\text{Customer, sum(O.Quantity*P.Price)}}$

$\bowtie_{\text{O.PID=P.PID}}$

Orders O

Products P

Customer	Sum
A	120
B	120
C	130
D	1

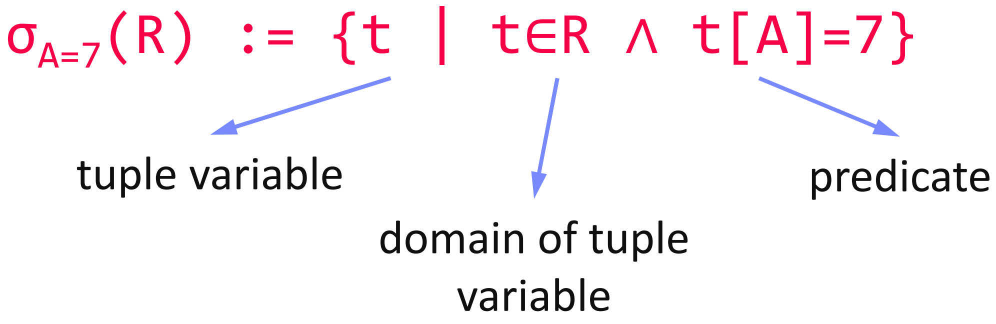
Tuple Calculus

Overview Tuple Calculus

- **Relational Calculus**
  - Tuple Calculus – tuple relational calculus: $\{T \mid p(T)\}$ (tuple → set)
    ➔ **examples:** see definition of relational algebra
  - (Domain Calculus – domain relational calculus: attribute → set)

- **Characteristics Tuple Calculus**
  - Calculus expression **does not specify order of operations**
  - Calculus expressions consist of variables, constants, comparison operators, logical concatenations, and quantifiers
  - Expressions are formulas, free formal variables → result

- **Example Selection**



$$\sigma_{A=7}(R) := \{t \mid t \in R \wedge t[A]=7\}$$

tuple variable

domain of tuple variable

predicate

Quantifiers

- **Variables**
  - Free: unbound variables define the result
  - Bound: existential quantifier **∃x** and universal quantifier **∀x** bind a variable x
- **Example Projection**

$$\pi_{A,B}(R) :=$$

$$\{t \mid \exists r \in R(t[A]=r[A] \land t[B]=r[B])\}$$

free variable **bound variable**

(relation T defined with two attributes A and B)

- **Safe Queries**
  - **Guarantees finite number of tuples** (otherwise, unsafe)
  - Example unsafe query: **{t | t ∉ R}**
  - **Relational completeness:** Every safe query expressible in RA and vice versa

Relational Algebra vs Tuple Calculus Revisited

- **E. F. Codd argued for Tuple Calculus**
  - **Criticism RA:** operator-centric
  - Ease of Augmentation (w/ lib functions)
  - Scope for Search Optimization
  - Authorization Capabilities
  - Closeness to Natural Language

→ **focus on query language**

[E. F. Codd: Relational Completeness of Data Base Sublanguages. IBM Research Report RJ987, **1972**]

- **System R Team used SEQUEL + RA**
  - **Criticism Tuple Calculus:** too complex
  - Iterating over tuples (not set-oriented)
  - Quantifiers and bound variables
  - Join over all variable attributes and result mapping

[Donald D. Chamberlin, Raymond F. Boyce: SEQUEL: A Structured English Query Language. SIGMOD Workshop **1974**]

- **Equivalent expressiveness + simplicity of RA + use as IR**
  → **Relational Algebra as basis for SQL und DBMS in practice**

Excursus: The History of System R and SQL

Gem: “The Birth of SQL – Prehistory / System R” (SQL Reunion 1995)

  - https://www.mcjones.org/System_R/SQL_Reunion_95/sqlr95-Prehisto.html
  - https://www.mcjones.org/System_R/SQL_Reunion_95/sqlr95-System.html

- **Don Chamberlin:** *We had this idea, that Codd had developed two languages, called the relational algebra and the relational calculus. […] The relational calculus was a kind of a strange mathematical notation with a lot of quantifiers in it. We thought that what we needed was a language that was* ***different from either one of those****, […].*
- **Don Chamberlin:** *Interestingly enough, Ted Codd didn't participate in that as much as you might expect. He got off into natural language processing […]. He really didn't get involved in the nuts and bolts of System R very much. I think he may have wanted to maintain a certain distance from it in case we didn't get it right. Which I think he would probably say we didn't.*
- **Mike Blasgen:** *Oh, he has said that, many times.*

Physical Operators

(Preview of Lecture 08 Query Processing)

Iterator Model

Scalable (small memory)
High CPI measures

- **Volcano Iterator Model**
  - Pipelined & no global knowledge
  - Open-Next-Close (ONC) interface
  - Query execution from root node (pull-based)

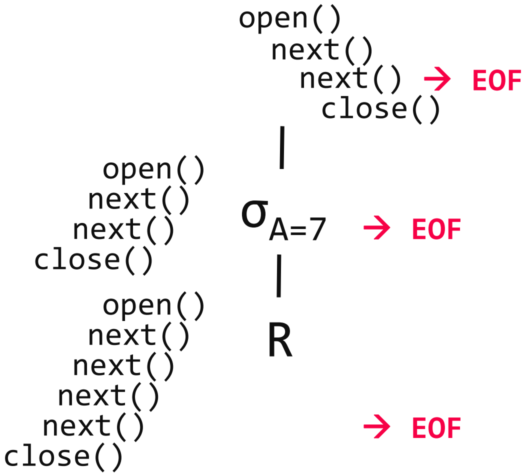
[Goetz Graefe: Volcano - An Extensible and Parallel Query Evaluation System. **IEEE Trans. Knowl. Data Eng. 1994**]

- **Example $\sigma_{A=7}(R)$**

```
void open() { R.open(); }

void close() { R.close(); }

Record next() {
  while( (r = R.next()) != EOF )
    if( p(r) ) //A==7
      return r;
  return EOF;
}
```

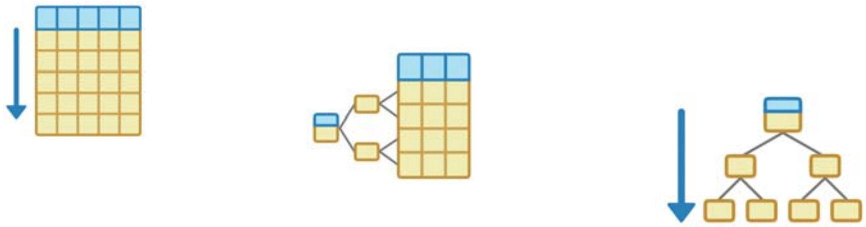


- **Blocking Operators**
  - Sorting, grouping/aggregation, build-phase of (simple) hash joins

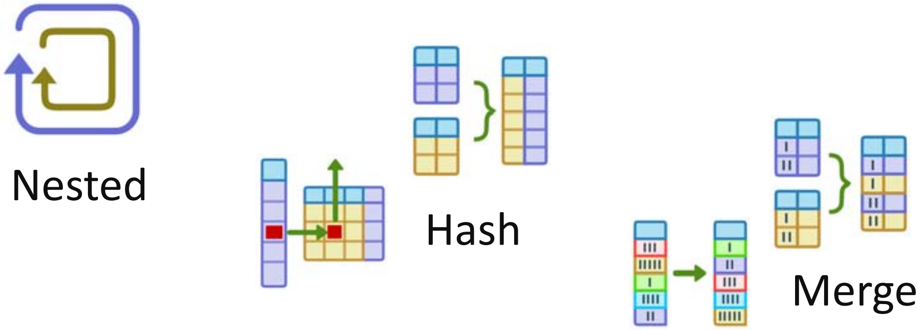
PostgreSQL: `Init()`, `GetNext()`, `ReScan()`, `MarkPos()`, `RestorePos()`, `End()`

(Selected) Physical Operators in PostgreSQL

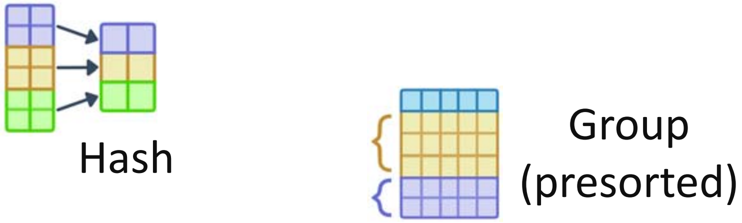
- **Physical Table Access Operators**
  - Seq Scan (table scan)
  - Index Scan
  - Index Only Scan



- **Physical Join Operators**
  - Nested Loop Join
  - Hash / Hash Join
  - Sort / Merge Join



- **Physical Grouping Operators**
  - Hash Aggregate
  - Group Aggregate



[**Images:** PostgreSQL/11/pgAdmin 4/web/pgadmin/misc/static/explain/img]

ANALYZE and EXPLAIN

Note: SQL for table creation and insert in the pptx notes.

- **Step 1:** `EXPLAIN SELECT * FROM Participant AS R, Locale AS S WHERE R.LID=S.LID;`

```
Hash Join (.. rows=70 width=1592)
  Hash Cond:(s.lid = r.lid)
  ->  Seq Scan on locale s (.. rows=140 width=520)
  ->  Hash (.. rows=70 width=1072)
        ->  Seq Scan on participant r (.. rows=70 width=1072)
```

build side

- **Step 2:** `ANALYZE Participant, Locale;`

- **Step 3:** `EXPLAIN SELECT * FROM Participant AS R, Locale AS S WHERE R.LID=S.LID;`

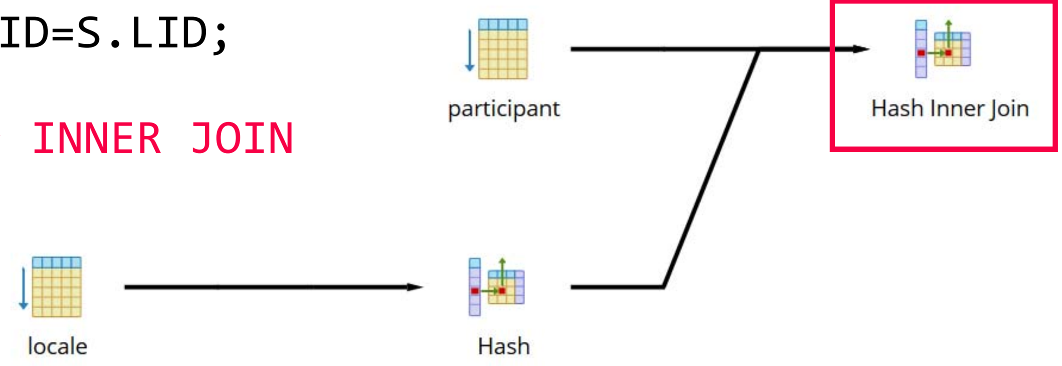
```
Hash Join (.. rows=17 width=47)
  Hash Cond:(r.lid = s.lid)
  ->  Seq Scan on participant r (.. rows=17 width=30)
  ->  Hash (.. rows=11 width=17)
        ->  Seq Scan on locale s (.. rows=11 width=17)
```

WHY?

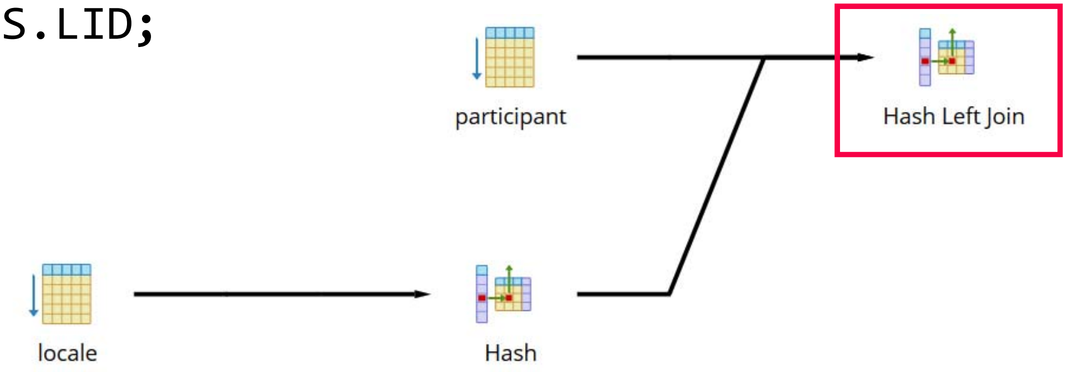
Visual EXPLAIN

- **SELECT** * **FROM** Participant **AS** R, Locale **AS** S
  WHERE R.LID=S.LID;

  $\sigma_F(R \times S)$ → INNER JOIN



- **SELECT** * **FROM** Participant **AS** R **LEFT JOIN** Locale **AS** S
  ON R.LID=S.LID;



Conclusions and Q&A

- **Summary**
  - Fundamentals of relational algebra and tuple calculus
  - Preview of query trees and physical operators

- **Exercise 1 Reminder**
  - All background to solve tasks 1.1-1.3 since last lecture
  - Submission: Deadline: **Nov 05 11.59pm**

- **Next Lectures**
  - Nov 04: **05 Query Languages (SQL)**, **incl. Exercise 2**
  - Nov 11: **06 APIs (ODBC, JDBC, OR frameworks)**