Programmierpraktikum: Datensysteme
02 Background Index Structures

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Last update: Oct 28, 2023

Announcements / Administrative Items

- **#1 Video Recording**
  - Hybrid lectures: in-person H 0111, zoom live streaming, video recording
  - https://tu-berlin.zoom.us/j/9529634787?pwd=R1ZsN1M3SC9BOU1OcFdmem9zT202UT09

- **#2 Team Selection**
  - Original: **Oct 23** team building/selection, afterwards assignment
  - Assignment: grouping of self-assignments (3 groups a 4 students), afterwards random assignment via `Team_ID = sample(35,35,FALSE) %% 9) + 3`
  - **Allowed post-deadline preferences** → **7 Java** Teams, **4 C/C++** Teams, **1 Rust** Team

Agenda

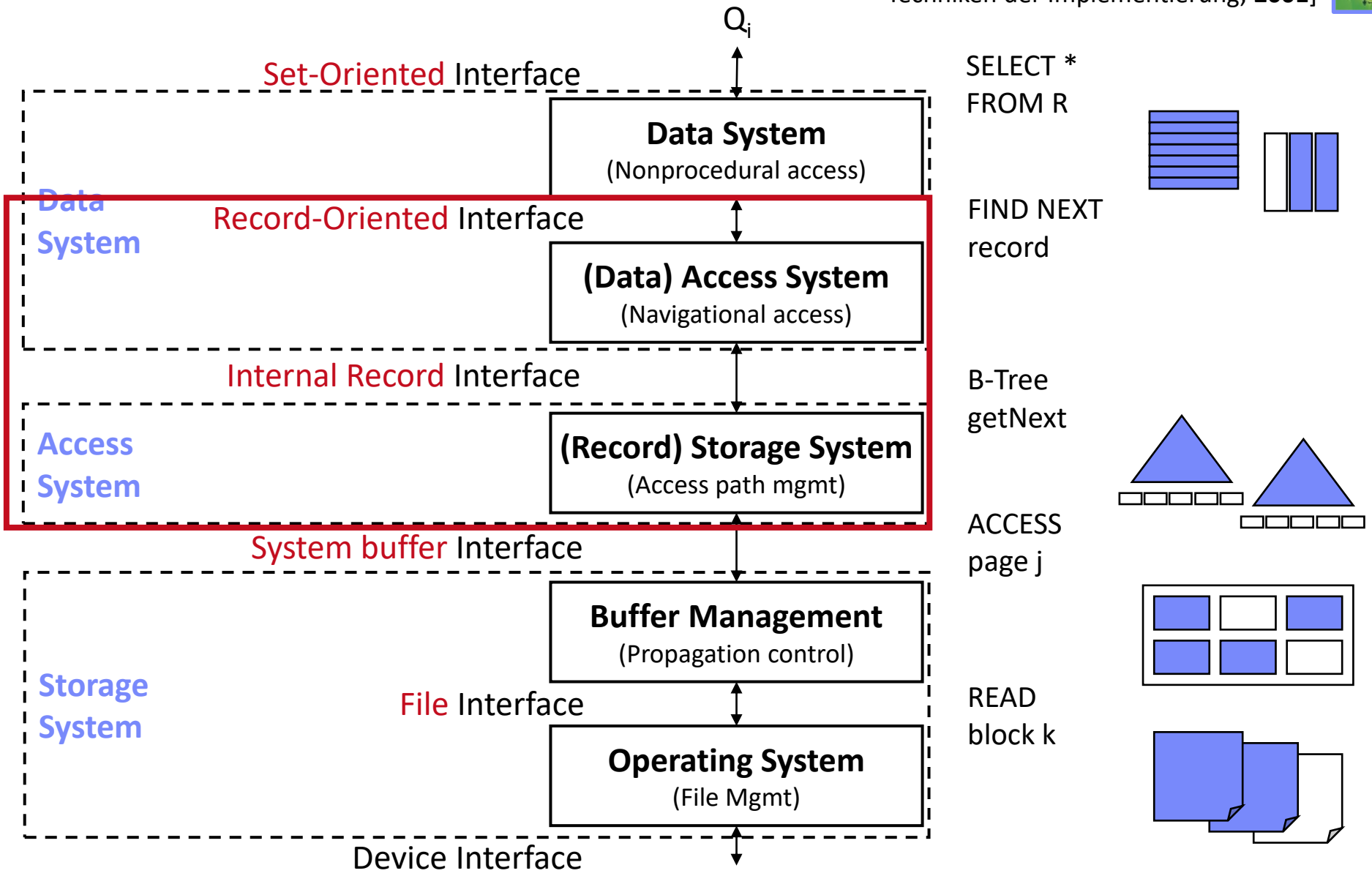
- **Overview Access Methods**
- **Index Structures**
- **Learned Indexing**

How to implement your own custom Index Structure

Overview Access Methods

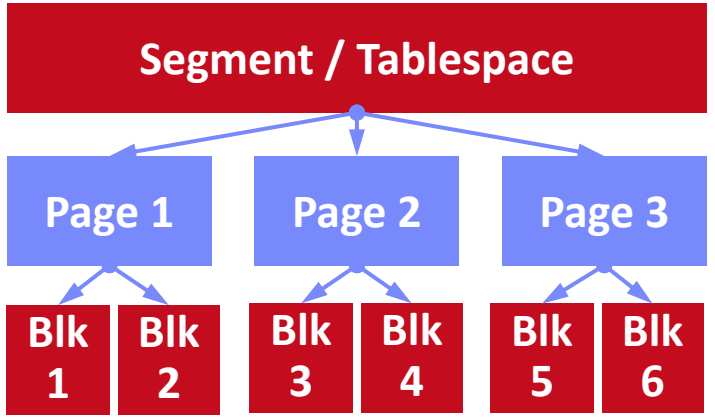
DBMS System Architecture

[Theo Härder, Erhard Rahm: Datenbanksysteme: Konzepte und Techniken der Implementierung, **2001**]

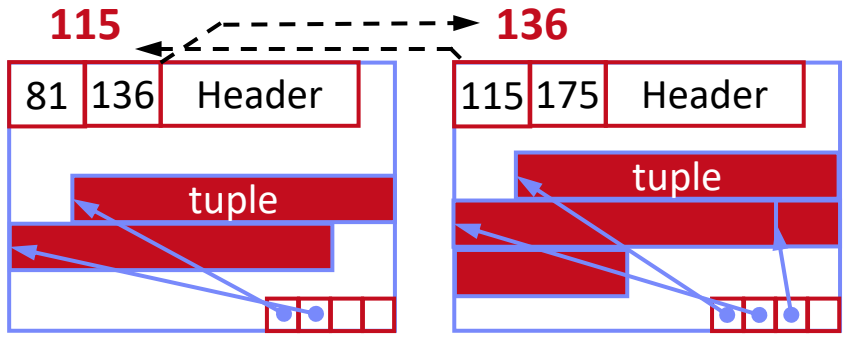


Background Storage System

- **Segments, Pages, Blocks**
  - **Segments:** storage unit of DB objects like relations (heap) and indexes
  - **Page:** fixed-size memory region
  - **Block:** smallest addressable unit on disk (e.g., POSIX block devices)



- **Buffer & Storage Management**
  - Buffer management at granularity of **pages**
  - PostgreSQL default: **8KB**
  - Different table/page layouts (e.g., **NSM**, DSM, PAX, column)
  - **TID/RID Concept** (pageID, slotID)
    → stable ID, even if **records reorganized**



Background Storage System, cont.

- **TID Concept (p, s)**
  - TID := (page number, slot index)
  - Page slot directory holds tuple offsets (byte position) within page

- **Example PostgreSQL**
  - Papers(PKey, Title, Pages, CKey, JKey)
  - Hidden CTID system column (not shown on *, but usable)

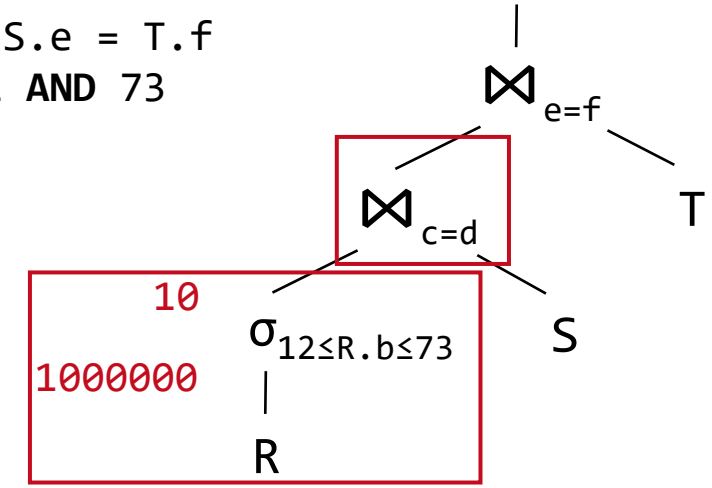
```
SELECT CTID, PKey,
       Title, Pages
  FROM Papers
```

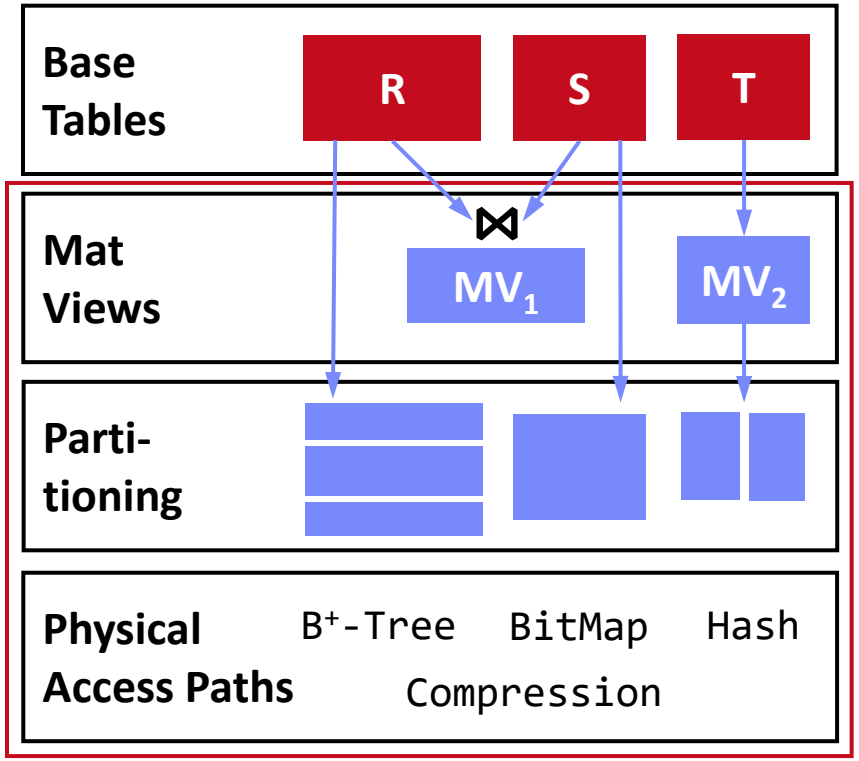
	ctid tid	pkey integer	title character varying (512)	pages character va
5681	(78,21)	731118	MV-IDX: Multi-Version Index in Action	671-674
5682	(78,22)	731121	Hochperformante Analyse von Graph-Dat...	311-330
5683	(78,23)	731122	SPARQling Pig - Processing Linked Data wi...	279-298
5684	(78,24)	731123	RelaX: A Webbased Execution and Learnin...	503-506
5685	(78,25)	731129	Efficient In-Memory Indexing with General...	227-246
5686	(78,26)	731130	Datensicherheit in mandantenfähigen Clo...	477-489
5687	(78,27)	731131	In-Database Machine Learning: Gradient ...	247-266
5688	(78,28)	731133	FlexY: Flexible; datengetriebene Prozessm...	503-506
5689	(78,29)	731134	Extending the MPSM Join	57-71
5690	(78,30)	731137	Orthogonal key-value locking	237-256

Access Methods and Physical Design

- **Performance Tuning via Physical Design**
  - Select physical data structures for relational schema and query workload
  - **#1:** User-level, **manual physical design** by DBA
  - **#2:** User/system-level **automatic physical design**
- **Example**

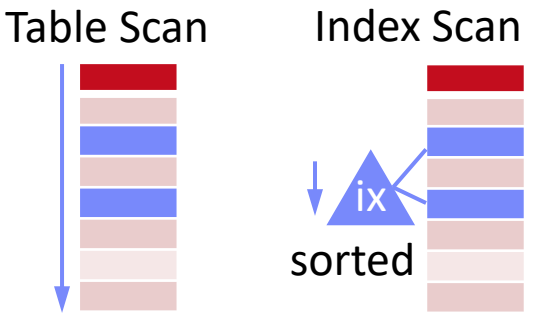
```
SELECT * FROM R, S, T
  WHERE R.c = S.d AND S.e = T.f
    AND R.b BETWEEN 12 AND 73
```





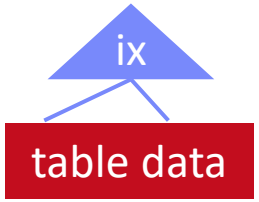
Overview Index Structures

- **Table Scan vs Index Scan**
  - For highly selective predicates, index scan **asymptotically much better** than table scan
  - Index scan **higher per tuple overhead** (break even ~5% output ratio)
  - Multi-column predicates: fetch/RID-list intersection

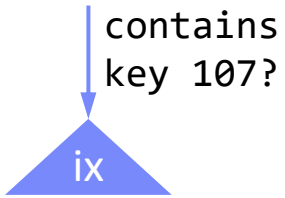


- **Use Cases for Indexes**

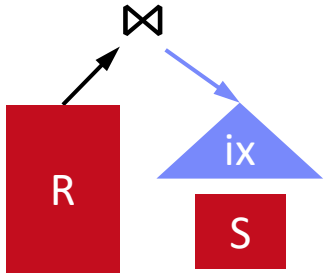
Lookups / Range Scans



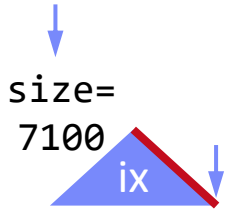
Unique Constraints



Index Nested Loop Joins



Aggregates (count, min/max)

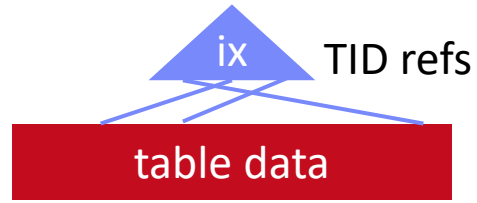


Additional Terminology

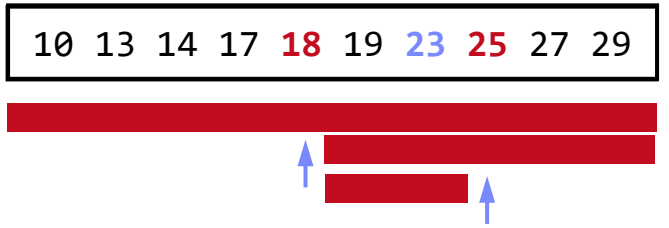
- **Create Index**
  - Create a secondary (non-clustered) index on a set of attributes
  - **Clustered:** tuples sorted by index
  - **Non-clustered:** sorted attribute with tuple references
  - Can specify uniqueness, order, and indexing method
  - **PostgreSQL methods:** btree, hash, gist, and gin

```
CREATE INDEX ixStudLname
  ON Students USING btree
  (Lname ASC NULLS FIRST);

DROP INDEX ixStudLname;
```



- **Binary Search**
  - `pos = binarySearch(data,key=23)`
  - Given **sorted data**, find key position (insert position if non-existing)
  - **k-ary search** for SIMD data-parallelism
  - **Interpolation search:** probe expected pos in key range (e.g., `search([1:10000], 9700)`)

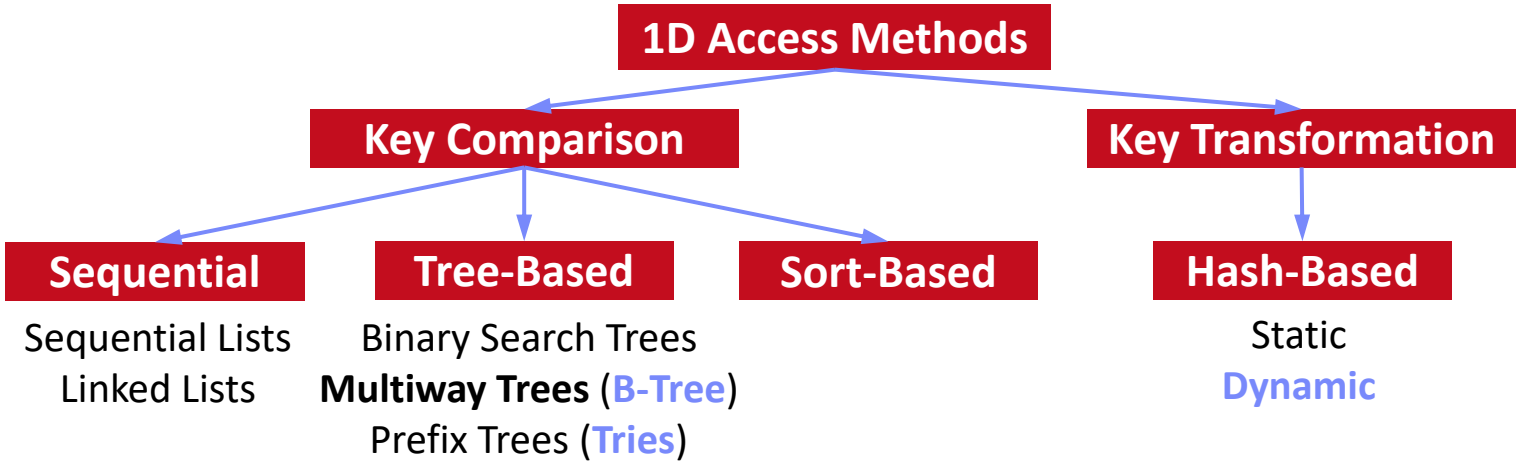


Index Structures

Classification of Index Structures

[Theo Härder, Erhard Rahm: Datenbanksysteme: Konzepte und Techniken der Implementierung, **2001**]

- **1D Access Methods**



- **ND Access Methods**
  - Linearization of ND key space + 1D indexing (Z order, Gray code, Hilbert curve)
  - Multi-dimensional trees and hashing (e.g., UB tree, k-d tree, gridfile)
  - Spatial index structures (e.g., R tree)

B-Tree Overview

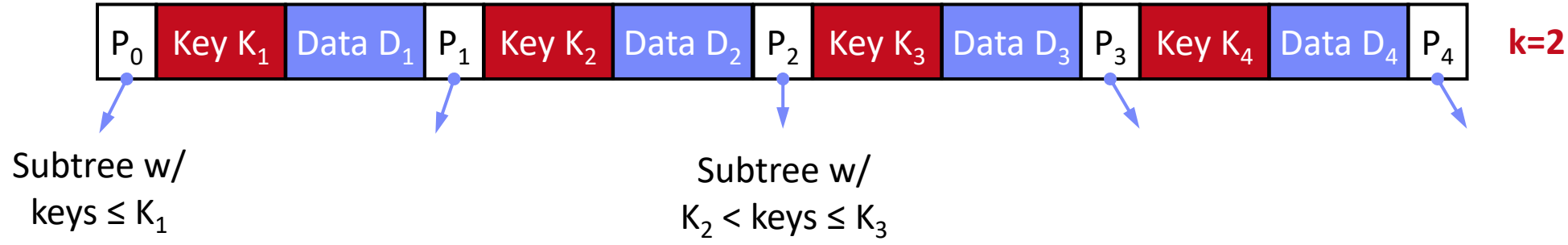
- **History B-Tree**
  - **B**ayer and McCreight 1972, **B**lock-based, **B**alanced, **B**oeing Labs
  - **Multiway tree** (node size = page size); designed for DBMS
  - Extensions: **B+-Tree/B*-Tree** (data only in leafs, double-linked leaf nodes)

[Rudolf Bayer, Edward M. McCreight: Organization and Maintenance of Large Ordered Indices. **Acta Inf. (1) 1972**]

- **Definition B-Tree (k, h)**
  - All paths from root to leafs have equal length h
  - All nodes (except root) have **[k, 2k]** key entries
  - All nodes (except root, leafs) have **[k+1, 2k+1]** successors
  - Data is a record or a reference to the record (RID)

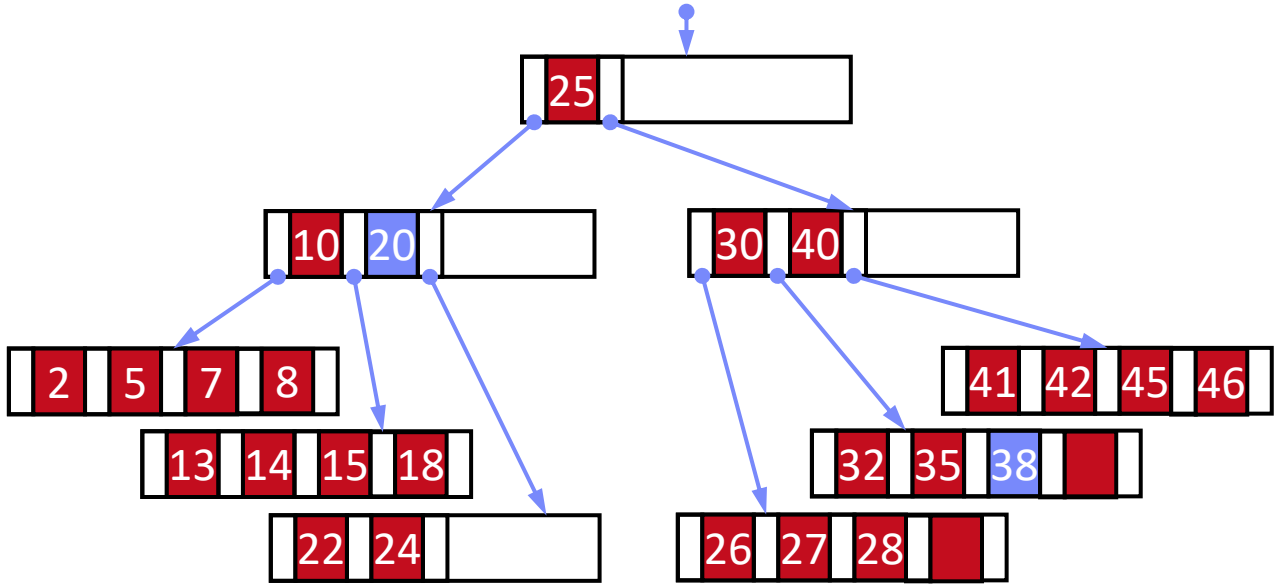
$$\lceil \log_{2k+1}(n+1) \rceil \leq h \leq \left\lceil \log_{k+1}\left(\frac{n+1}{2}\right) \right\rceil + 1$$

All nodes adhere to max constraints



B-Tree Search

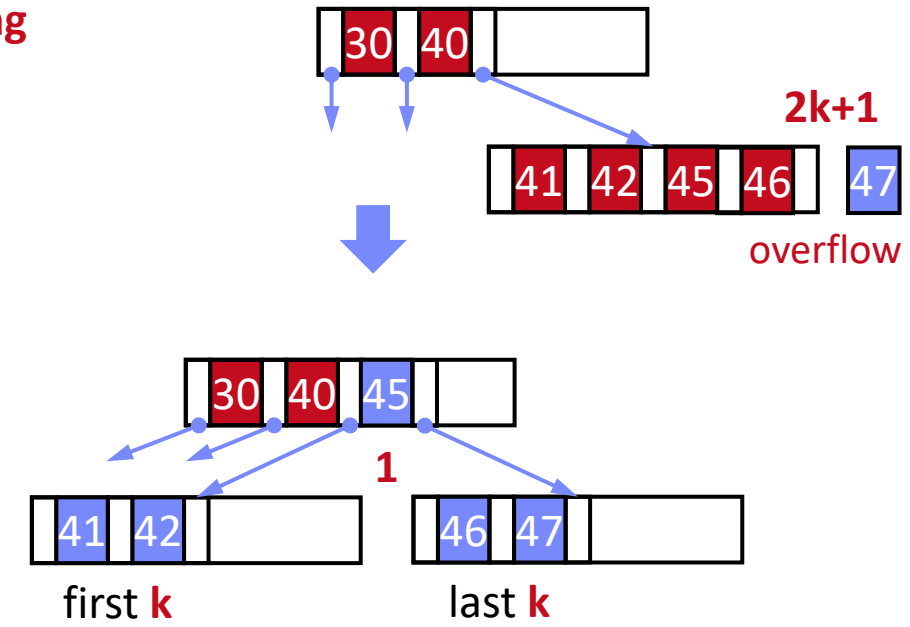
- **Example B-Tree k=2**
  - Get 38 → D38
  - Get 20 → D20
  - Get 6 → NULL



- **Lookup Q_K within a node**
  - Scan / binary search keys for Q_K, if $K_i=Q_K$, return D_i
  - If node does not contain key
    - If leaf node, abort search w/ NULL (not found), otherwise
    - Decent into subtree Pi with $K_i < Q_K \leq K_{i+1}$
- **Range Scan $Q_{L<K<U}$**
  - Lookup Q_L and call next K while $K<Q_U$ (keep current position and node stack)

B-Tree Insert

- **Basic Insertion Approach**
  - **Always insert into leaf nodes!**
  - Find position similar to lookup, insert and maintain sorted order
  - If node overflows (exceeds 2k entries) ➔ **node splitting**
- **Node Splitting Approach**
  - Split the **2k+1** entries into two leaf nodes
  - **Left node:** first k entries
  - **Right node:** last k entries
  - (k+1)th entry inserted into parent node
    ➔ can cause **recursive splitting**
  - Special case: root split (h++)
- **B-Tree is self-balancing**

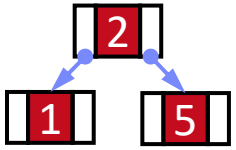


B-Tree Insert, cont. (Example w/ k=1)

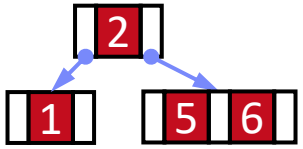
- Insert 1

- Insert 5

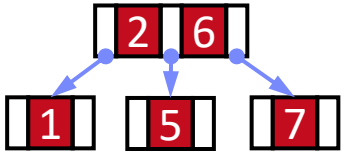
- Insert 2 (split)



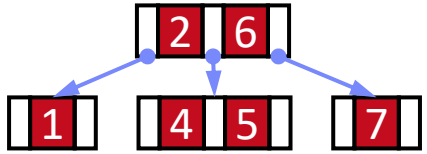
- Insert 6



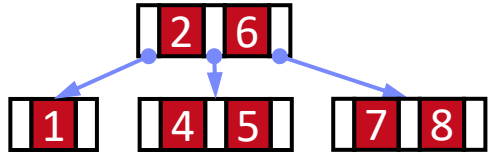
- Insert 7 (split)



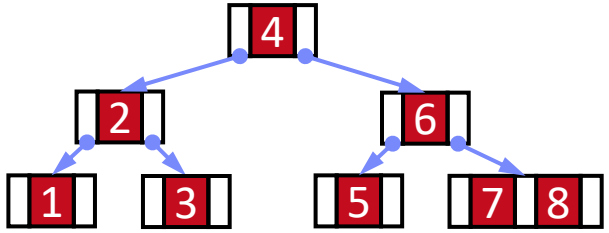
- Insert 4



- Insert 8



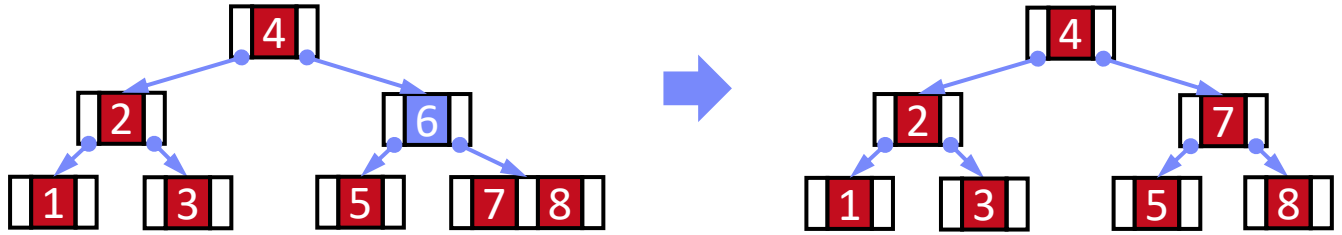
- Insert 3 (2x split)



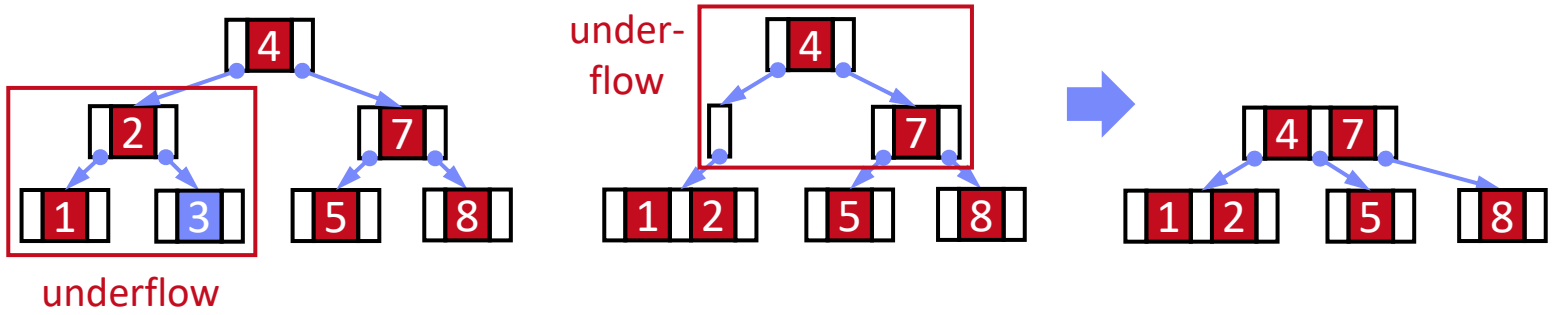
B-Tree Delete

- **Basic Deletion Approach**
  - Lookup deletion key, abort if non-existing
  - Case inner node: **move entry** from fullest successor node into position
  - Case leaf node: if underflows (<k entries) ➔ **merge w/ sibling**

- **Example**
  - Case inner

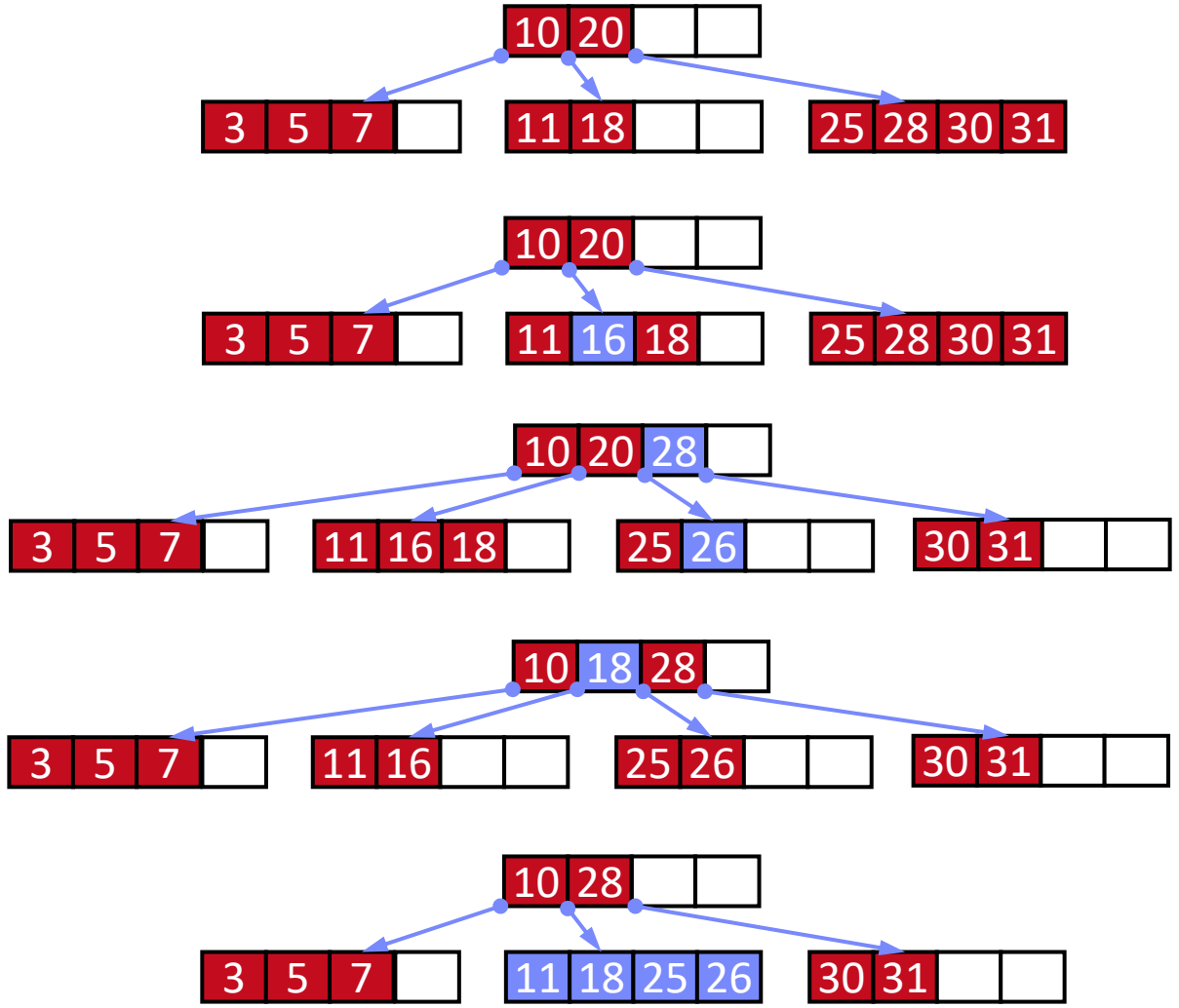


  - Case leaf



B-Tree Insert and Delete w/ k=2

- **Insert/Delete Examples**
  - Original
  - Insert 16
  - Insert 26
  - Delete 20
  - Delete 16



B-tree – Advanced Aspects

[Goetz Graefe: Modern B-Tree Techniques. **Found. Trends Databases** 3(4): 203-402, **2011**]

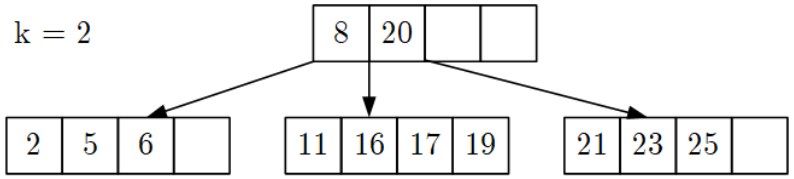
- **Variable-Length Fields**
  - In-page slot-array to variable length fields → direct lookup
  - With fixed page size, **no guarantees on min/max entries**
  - **Various approaches:** overflow pages, pick separators during bulk loading

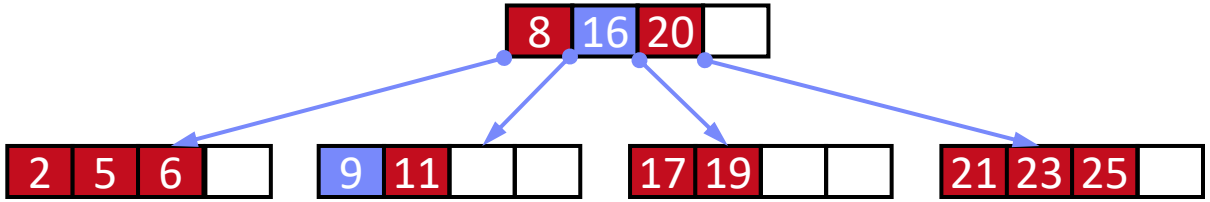
- **Concurrent Access**
  - DB locks: only leaf nodes for B+ tree in practice at **value/value ranges**
  - Concurrent threads require page latching (parent-child)

- **Duplicate Keys**
  - **#1** use **prefix truncation** for compression → store common prefix once)
  - **#2 concatenate key-TID** for unique lockups w/ O(log N)
  - Duplicate records as replicates or once w/ counter

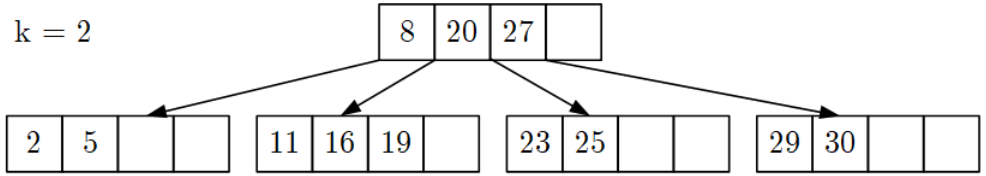
TEST YOURSELF: B-tree Understanding

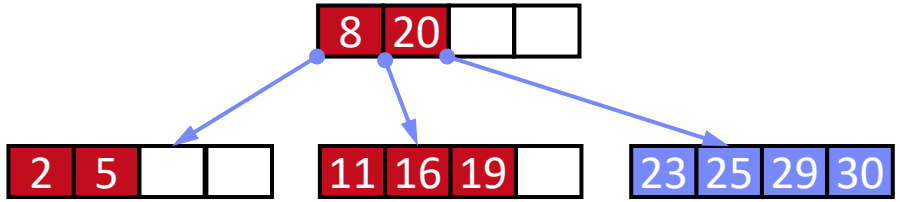
- **Given B-tree below, insert key 9 and draw resulting B-tree** (5/100 points)





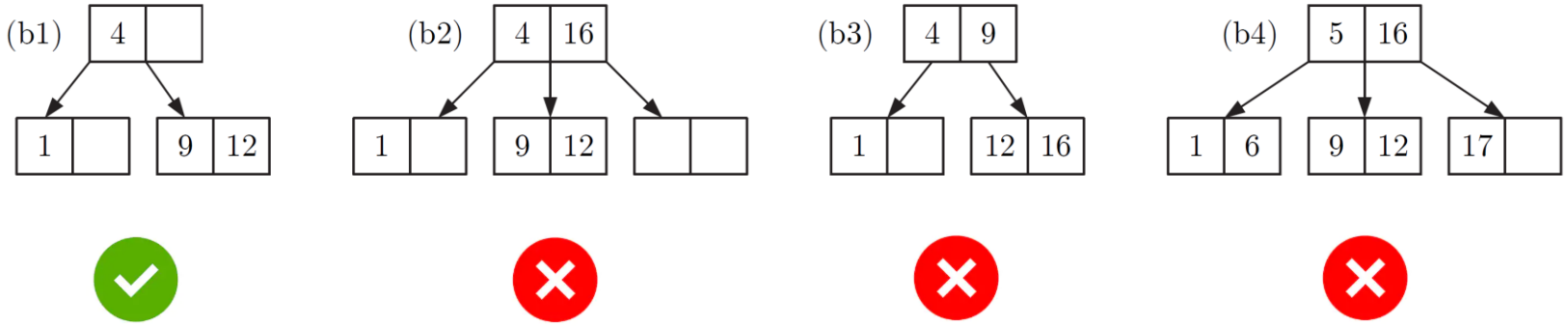
- **Given B-tree below, delete key 27, and draw resulting B-tree** (5/100 points)





TEST YOURSELF: B-tree Understanding, cont.

- **Which of the following trees are valid – i.e., satisfy the constraints of – B-trees with k=1? Mark each tree as valid or invalid and name the violations** (4/100 points)



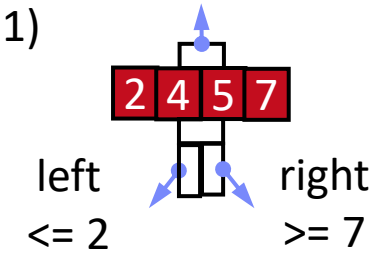
(empty leaf node, **underflow**)

(**invalid #** of pointers and subtrees)

(**invalid ordering** of data items, 6>5 but in left subtree)

Other In-Memory Trees

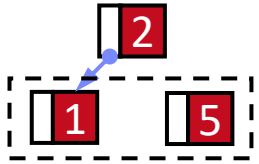
- **Balanced Binary Trees**
  - Red-Black Tree, AVL Tree (left/right height diff 1)
  - T tree (combines pros of AVL and B trees)



[G. M. Adel'son-Vel'skii and E. M. Landis: An algorithm for the organization of information, Soviet Mathematics Doklady, 3, **1962**]

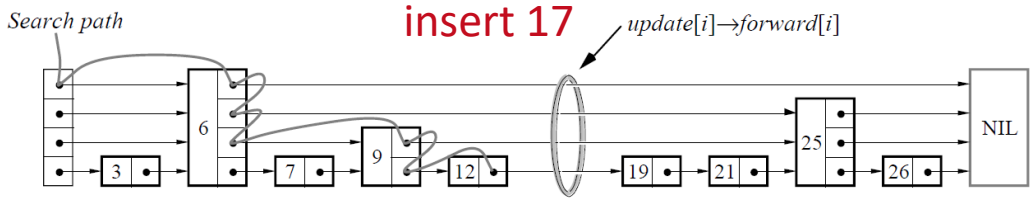
[Tobin J. Lehman, Michael J. Carey: A Study of Index Structures for Main Memory Database Management Systems. **VLDB 1986**]

- **CSB$^+$-Tree**
  - Align node size to cache line (64B)
  - Reduce pointers via node groups
  - More keys, higher fan-out, at cost of slower insert



[Jun Rao, Kenneth A. Ross: Making B+-Trees Cache Conscious in Main Memory. **SIGMOD 2000**]

- **Skip Lists**
  - Linked list w/ multiple levels
  - Fractions p w/ pointers

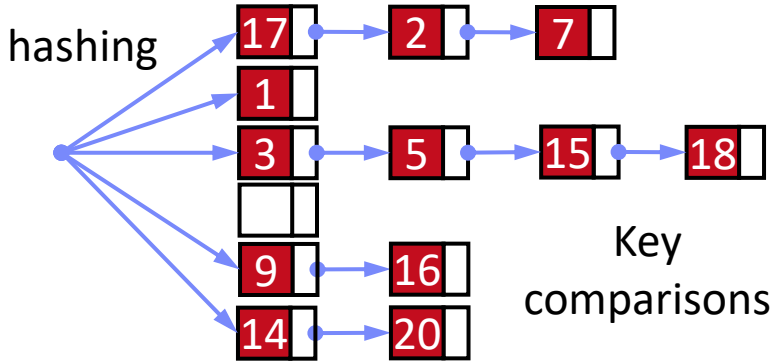


[William Pugh: Skip Lists: A Probabilistic Alternative to Balanced Trees. **CACM 1990**]

Hashing Overview

- **Static vs Dynamic Hashing**
  - Hash table of buckets B, compute h=hash(key), find bucket B[h mod |B|]
  - **Static:** pre-allocation of buckets, **over- and under-provisioning** (open addressing: linear probe, robin hood, cuckoo)
  - **Dynamic:** extend as needed (chained bucket, extendible, linear)

- **Chained Bucket Hashing**
  - Handle hash collisions via **overflow list** of linked buckets
  - Reorganization if fill factor reached
  - **On disk:** buckets are pages



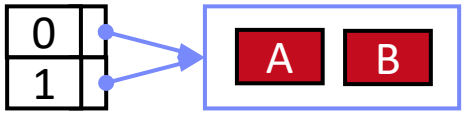
- **Common Hash Functions**
  - MurmurHash 2, MurmurHash 3, Jenkins, CRC
  - Google CityHash, Google FarmHash, Facebook XXHash3 (http://cyan4973.github.io/xxHash/)

[Andy Palvo: Database Systems – Hash Tables, CMU Lecture, **2019**]

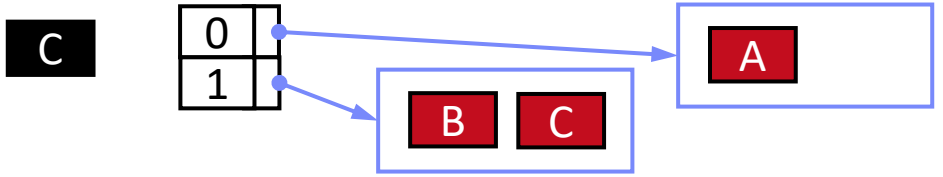
Extendible Hashing

[Ronald Fagin, Jürg Nievergelt, Nicholas Pippenger, H. Raymond Strong: Extendible Hashing - A Fast Access Method for Dynamic Files. **TODS 4(3), 1979**]

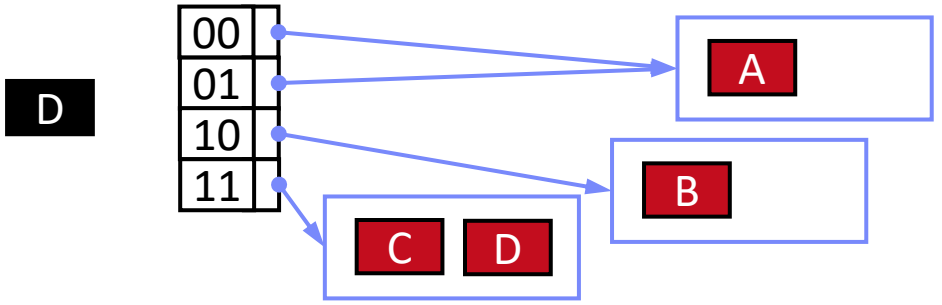
- **Overview**
  - Dynamic resizing on demand, w/o rehashing/reassigning tuples to pages
  - h=hash(key), use **d bits** and **directory of 2^d entries** (with max table size, then bucket chaining)
  - Directory entries point to buckets, multiple refs to one bucket possible



- **Example d = 1**



- **Example d = 2**

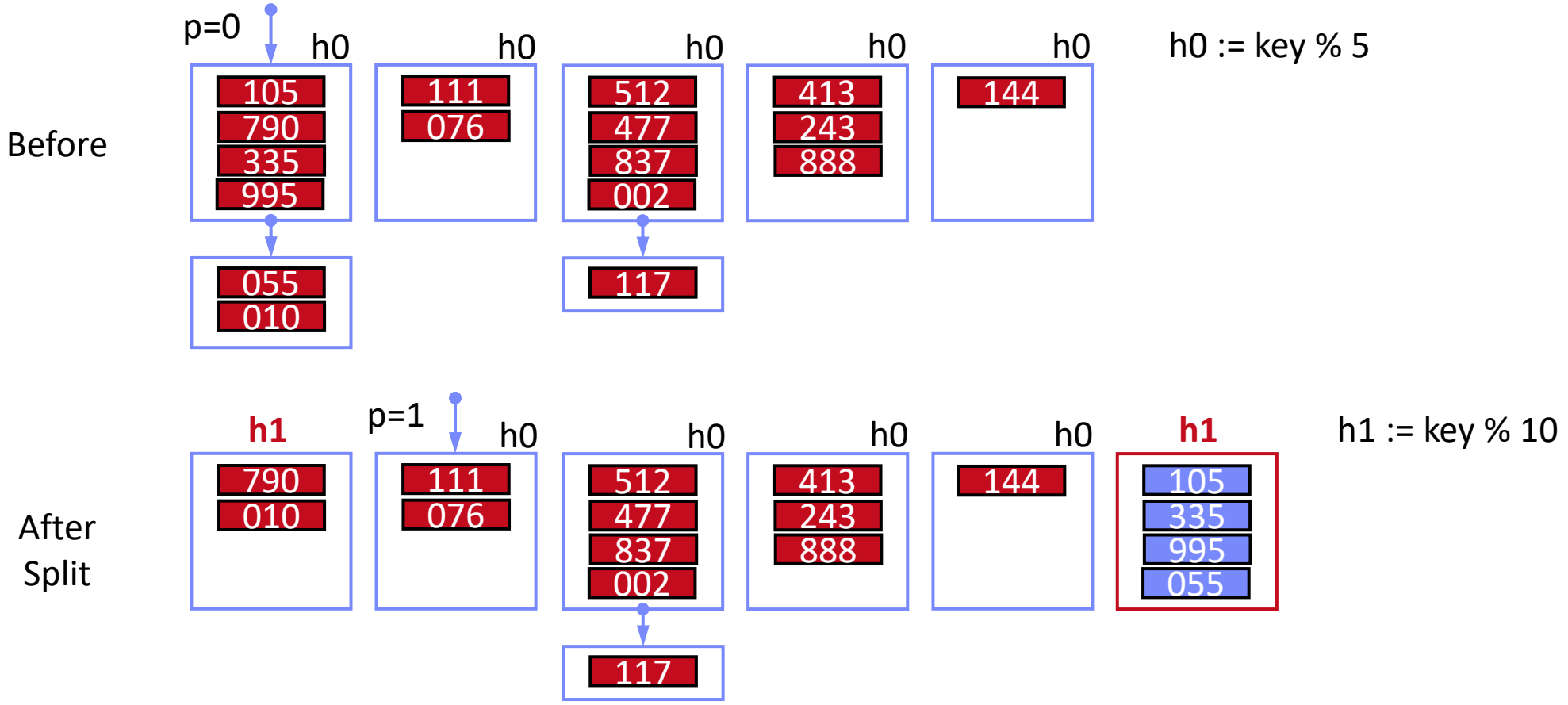


[Thomas Neumann: Datenbanksysteme und moderne CPU-Architekturen – Access Paths, TU Munich, **2019**]

Linear Hashing

[Theo Härder, Erhard Rahm: Datenbanksysteme: Konzepte und Techniken der Implementierung, **2001**]

- **Overview**
  - Improved Extensible Hashing scheme, w/o exponential directory growth
  - First start chaining, then incrementally **split individual buckets** (in order)
- **Example**

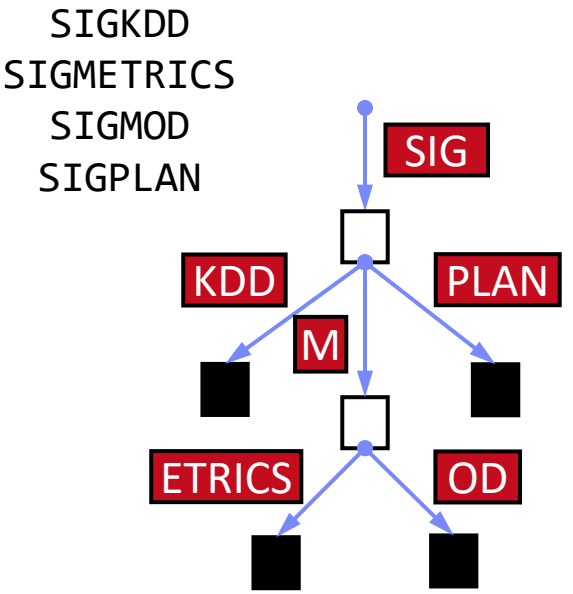


Overview Prefix Trees (Tries)

- **Overview**
  - From information re**trie**val, mostly for string indexing
  - **Trie:** "A tree for storing strings in which there is one node for every common prefix. The strings are stored in extra leaf nodes." (NIST DADS)

- **PATRICIA Trie**
  - Extended binary (character-level) trie, with compressed substrings

[Donald R. Morrison: PATRICIA - Practical Algorithm To Retrieve Information Coded in Alphanumeric. **J. ACM 15(4) 1968**]



- **Variants**
  - Radix Tree, key alteration radix tree (Kart), digital search trees

Generalized Prefix Tree

[Matthias Boehm et al: Efficient In-Memory Indexing with Generalized Prefix Trees. **BTW 2011**]

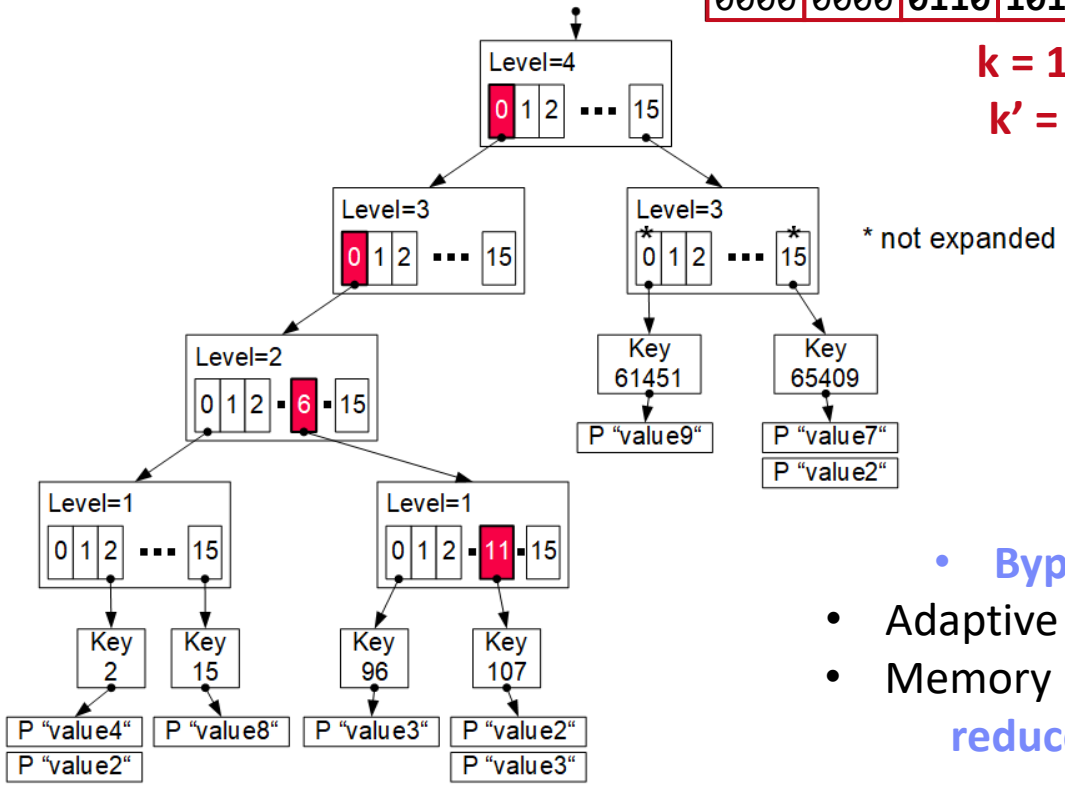
- **Generalized Prefix Tree**
  - Arbitrary data types (byte sequences)
  - Configurable prefix length **k'**
  - Node size: **$s = 2^{k'}$** references
  - Fixed maximum height **h = k/k'**
  - Secondary index structure
- **Characteristics**
  - Partitioned data structure
  - Order-preserving (for range scans)
  - Update-friendly
- **Properties**
  - **Deterministic paths**
  - Worst-case complexity O(h)

insert (**107**,value4)

0000	0000	**0110**	**1011**

k = 16

k' = 4



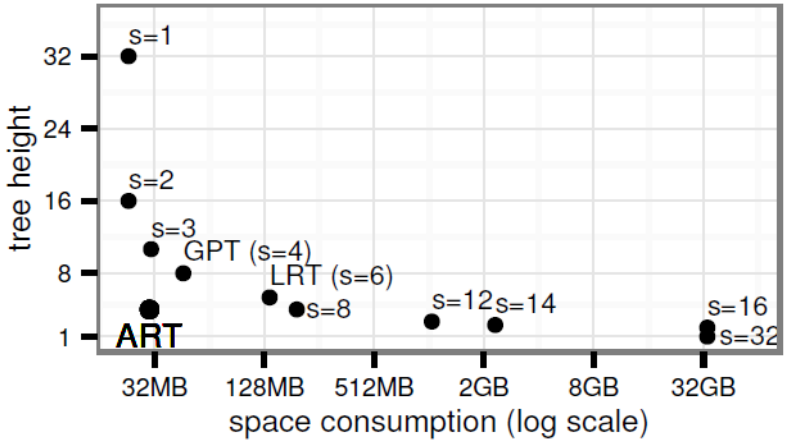
Node size (k'=4):
128B → 64B

- **Bypass array**
- Adaptive **trie expansion**
- Memory preallocation + **reduced pointers**

Adaptive Radix Trees

[Viktor Leis, Alfons Kemper, Thomas Neumann: The adaptive radix tree: ARTful Indexing for Main-Memory Databases. **ICDE 2013**]

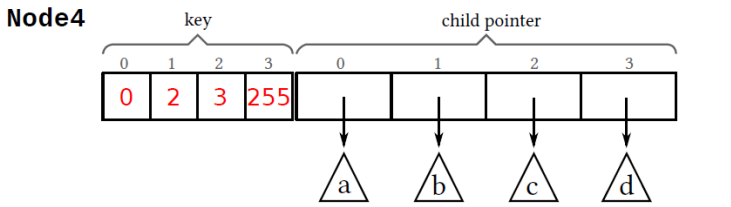
- **Motivation and Overview**
  - Small trie height/high fan-out, but with low space overhead
  - **Prefix k'=8** → 256 children
  - Adaptive nodes 4, 16, 48, 256 entries
  - **Lazy expansion** and **path compression**

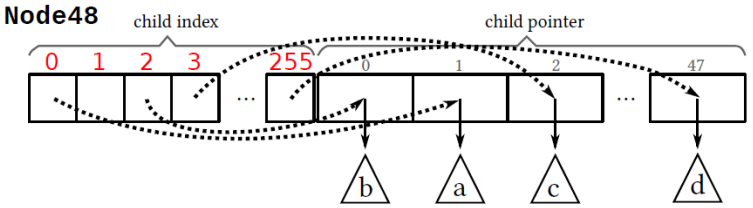


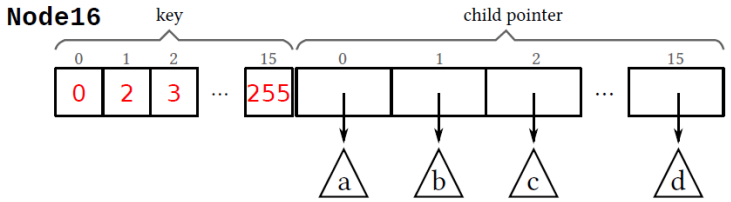
- **Node Types**

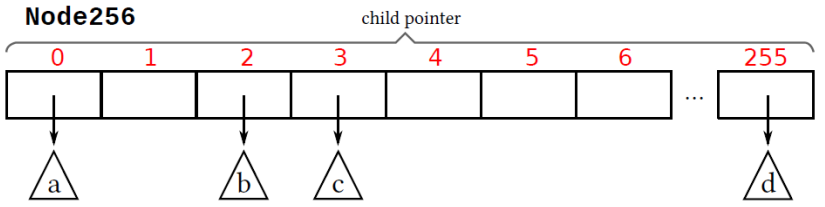
Linear/binary search for keys

256 element arrays of indexes / child pointers









Hybrid Prefix Trees

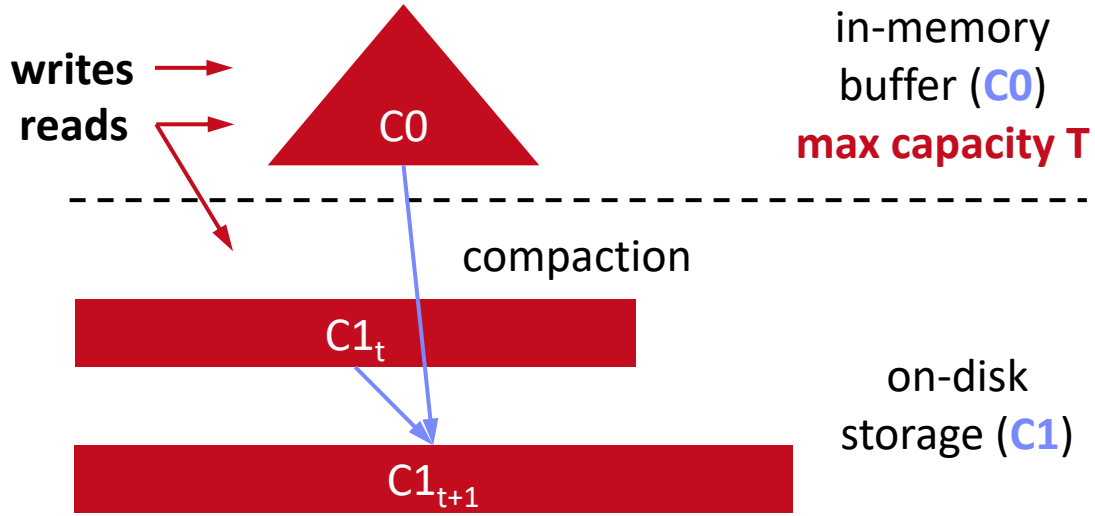
	Binary	B-Tree	CSB-Tree	Hash	T-Tree	Trie
Prefix Hash Tree ’70				X		X
Prefix B-Tree ’77		X				X
Ternary Search Tree ’97	X					X
Partial Keys ’01		X			X	X
Burst-Trie ’02	X	X	X	X	X	X
HAT-Trie ’07				X		X
J$^+$-Tree ’09		X			X	X
CS-Prefix Tree ’09			X			X
SuRF ’18				X		X

Log-structured Merge Trees (LSM)

[Patrick E. O'Neil, Edward Cheng, Dieter Gawlick, Elizabeth J. O'Neil: The Log-Structured Merge-Tree (LSM-Tree). **Acta Inf. 1996**]

- **LSM Overview**
  - Many KV-stores rely on LSM-trees as their storage engine (e.g., BigTable, DynamoDB, LevelDB, Riak, RocksDB, Cassandra, HBase)
  - **Approach:** Buffers writes in memory, flushes data as sorted runs to storage, merges runs into larger runs of next level (compaction)

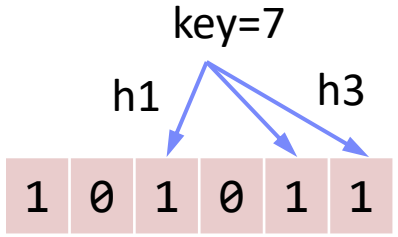
- **LSM System Architecture**
  - Writes in C0
  - Reads against C0 and C1 (w/ buffer for C1)
  - Compaction (rolling merge): sort, merge, incl deduplication



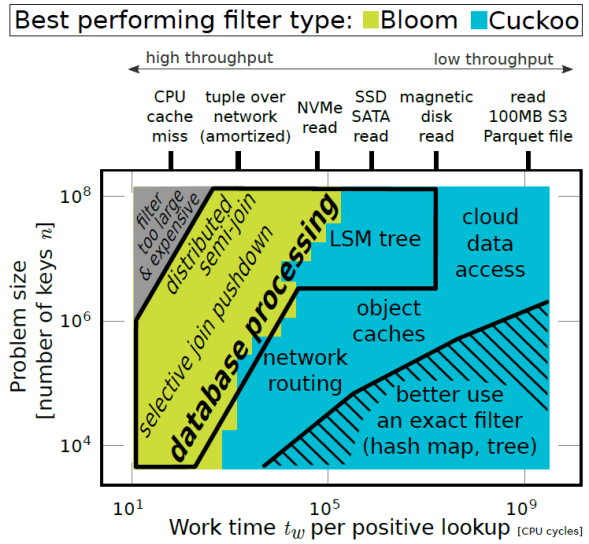
Probabilistic Set Containment

- **Motivation**
  - Many use cases for applying cheap pre-filters
  - **Requirement: no false negatives**, small number of false positives (FP)

- **#1 Bloom Filter**
  - Array X of m bits, initialized w/ zeros
  - k different hash functions applied on each key
  - **Insert:** k x h_i(key), set all hashed positions to 1
  - **Query:** k x h_i(key), return (sum(X[h_i(key)])==k)



- **#2 Cuckoo Filter**
  - Cuckoo hash table with key signatures
  - 2 hash functions w/ displacements
  - Allows deletes, duplicates, smaller FP rate



[Harald Lang, Thomas Neumann, Alfons Kemper, Peter A. Boncz: Performance-Optimal Filtering: Bloom overtakes Cuckoo at High-Throughput. **PVLDB 12(5), 2019**]

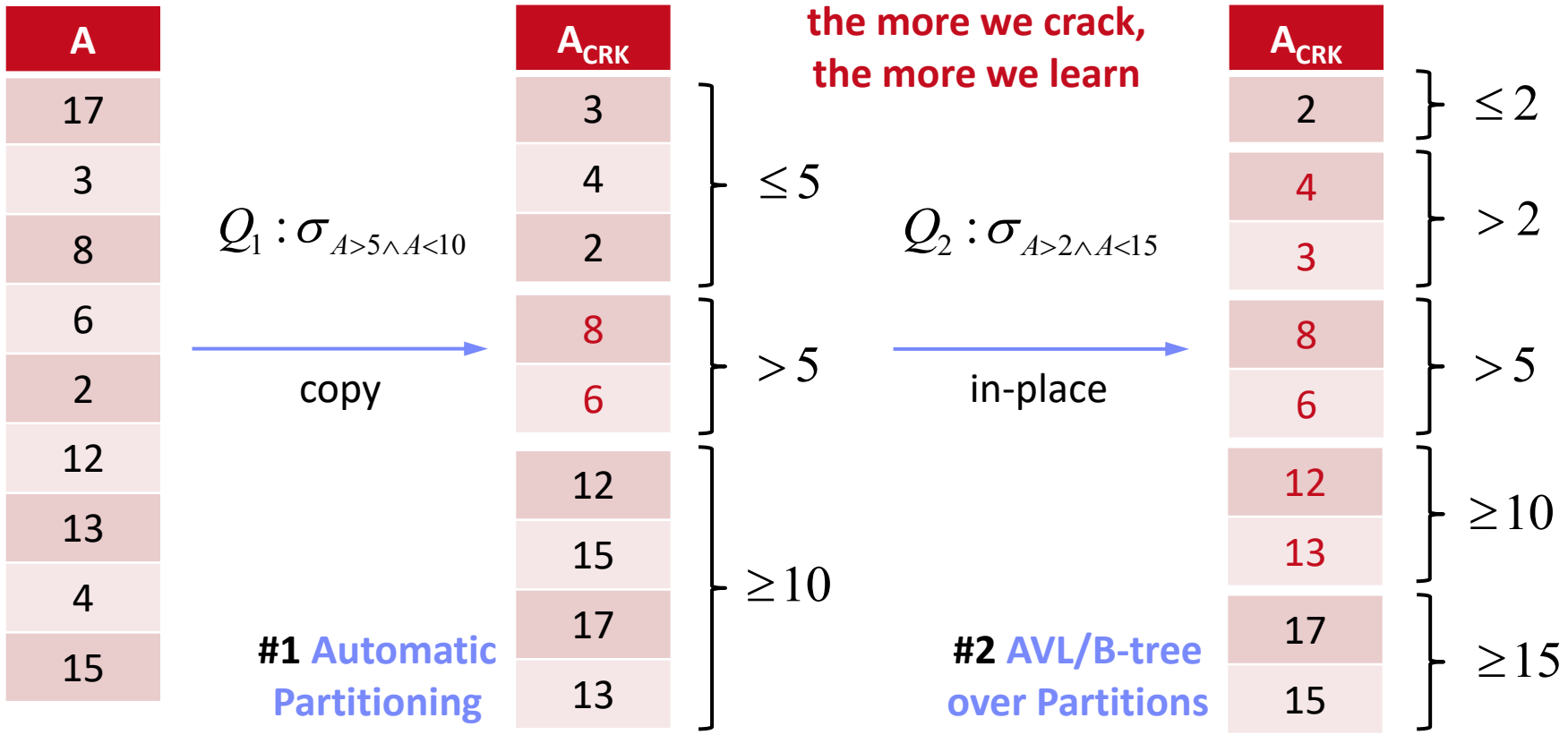
Learned Indexing

Excursus: Database Cracking

[Pedro Holanda et al: Progressive Indexes: Indexing for Interactive Data Analysis. **PVLDB 2019**]

[Stratos Idreos, Martin L. Kersten, Stefan Manegold: Database Cracking. **CIDR 2007**]

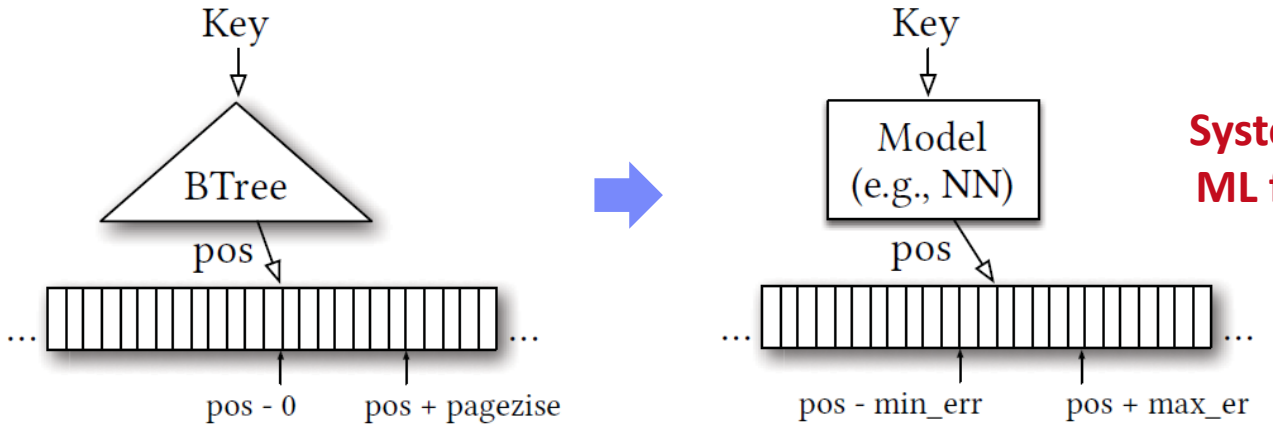
- **Core Idea:** Queries trigger physical reorganization (partitioning and indexing)



Learned Index Structures

[Tim Kraska, Alex Beutel, Ed H. Chi, Jeffrey Dean, Neoklis Polyzotis: The Case for Learned Index Structures. **SIGMOD 2018**]

- **A Case For Learned Index Structures**
  - Sorted data array, predict position of key
  - **Hierarchy of simple models** (stages models)
  - Tries to **approximate the CDF** similar to interpolation search (uniform data)



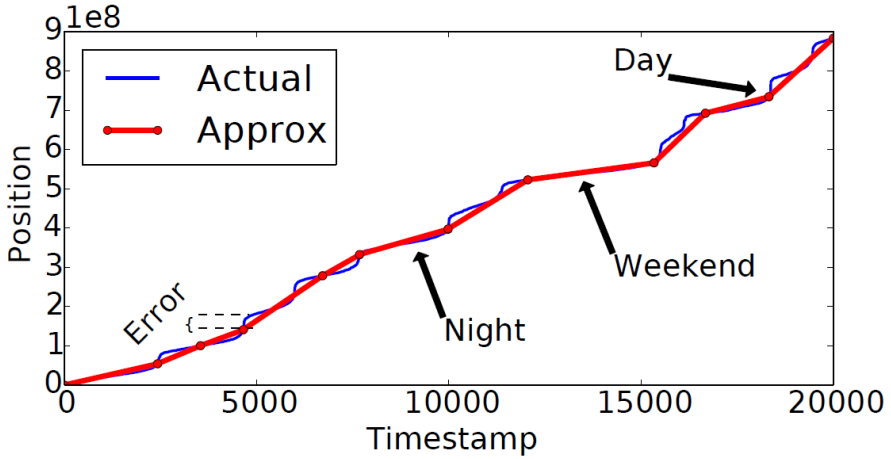
Systems for ML, ML for Systems

- **Follow-up Work on SageDBMS**

[Tim Kraska, Mohammad Alizadeh, Alex Beutel, Ed H. Chi, Ani Kristo, Guillaume Leclerc, Samuel Madden, Hongzi Mao, Vikram Nathan: SageDB: A Learned Database System. **CIDR 2019**]

Learned Index Structures, cont.

- **FITing-Tree**
  - Adapt to underlying data and patterns
  - Piecewise linear functions
  - Maximum pos error guarantees
  - Segment pages w/ free space



[Alex Galakatos, Michael Markovitch, Carsten Binnig, Rodrigo Fonseca, Tim Kraska: FITing-Tree: A Data-aware Index Structure. **SIGMOD 2019**]

- **PGM-index**
  - Piecewise geometric model index
  - Recursive, compressed segment tree

[Paolo Ferragina, Giorgio Vinciguerra: The PGM-index: a fully-dynamic compressed learned index with provable worst-case bounds. **PVLDB 13(8) 2020**]

- **RadixSpline**
  - Lookup table to spline points, selected w/ max error guarantee

[Andreas Kipf, Ryan Marcus, Alexander van Renen, Mihail Stoian, Alfons Kemper, Tim Kraska, Thomas Neumann: RadixSpline: a single-pass learned index. **aiDM@SIGMOD 2020**]

Summary & QA

- **Overview Access Methods**
- **Index Structures**
- **Learned Indexing**

- **Next Lectures**
  - Nov 13: **Background Transaction Processing**
  - Nov 27: **Experiments and Reproducibility**
  - Additional lectures / Q&A sessions on demand

Thanks